Multiresolution methods for unstructured data 2025

Program

Program

	30.09.25
17:45–18:00	**Registration and Welcome**
18:00–20:00	*Aperitivo*

	01.10.25	02.10.25
09:00–09:40	**M. Maggioni** Multiscale Geometric Methods for high-dimensional data near low-dimensional sets	**J. Tausch** On the Computation of Singular Integrals over Convex Polytopes
09:40–10:20	**J. Dölz** Data-intrinsic approximation in metric spaces	**H. Harbrecht** Kernel interpolation on generalized sparse grids
10:20–11:00	*Coffee Break*	
11:00–11:40	**W. Erb** Graph Wedgelets: adaptive decomposition of graph signals based on wedge partitioning trees and geometric wavelets	**H. Wendland** Multiscale Approximation by Radial Basis Functions
11:40–12:20	**M. Bulai** Wavelet packets and graph signal processing	**T. Wenzel** Multiscale greedy kernel algorithms
12:20–14:00	*Lunch*	
14:00–14:40	**P. Balazs** Frame Theory: the mathematical foundation for acoustics, quantum physics, numerics and machine learning	**A. Iske** On the convergence of multiscale kernel regression under minimalistic assumptions
14:40–15:20	**D. Peterseim** Multilevel Preconditioning on Quantum Computers	**G. Santin** Superconvergence in Kernel Interpolation: A General Framework and Algorithmic Implications
15:20–16:00	*Coffee Break*	
16:00–16:40	**R. Abgrall** Multiresolution on unstructured meshes	
19:00–22:00	*Social dinner at Grotto della Salute*	

Abstracts

Multiresolution on unstructured meshes

Rémi Abgrall (University of Zurich)

Co-authors:
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Abstract

In this paper, I will describe techniques to represent data which originate from discretization of functions in unstructured meshes in terms of their local scale components. To do so, I consider a nested sequence of discretization, which corresponds to increasing levels of resolution, and we define the scales as the ”difference in information” between any two successive levels. We obtain data compression by eliminating scale-coefficients which are sufficiently small. This capability for data compression can be used to reduce the cost of numerical schemes by solving for the more compact representation of the numerical solution in terms of its significant scale-coefficients. This method is an extension to unstructured meshes of the technique developed by Ami Harten in several publications in the 1990s. I will also show applications to compressible fluid dynamics, explain what, in my opinion, the limits of the method are, and how they could possibly be removed. This work had been conducted mostly with A. Harten; see [1].

References

[1] R. Abgrall, and A. Harten, *Multiresolution representation in unstructured meshes*, SIAM J. Numer. Anal., 35(6):2128–2146, 1998.

Frame Theory: the mathematical foundation for acoustics, quantum physics, numerics and machine learning

Peter Balazs (Austrian Academy of Science)

Abstract

In this overview talk, we give a broad reflection of frame theory and the connection to various application areas, in particular acoustics. We will introduce the basic definition and concept of frames. We will link them to the concept of coherent states in quantum physics. We will talk about time-frequency analysis and its link to frame theory. As a particular form of quantization operators, we will present frame multipliers—operators that can be represented as a weighted version of the frame decomposition. We will show how they are applied in signal processing as time-variant filters. We will introduce the representation of operators using frames, and show the link to numerical approaches like FEM/BEM and the applications to acoustical simulations. We will present sound signals of a particular application in acoustics: audio inpainting. Finally, we will hint at the connection of frames to deep neural networks.

Wavelet packets and graph signal processing

Martina Bulai (University of Turin)

Co-authors:
Sandra Saliani

Abstract

Nowadays, graphs have become of significant importance given their use to describe complex system dynamics, with important applications to real-world problems, e.g., graph representation of the brain, social networks, biological networks, spreading of a disease, etc. In this work [4], we introduce a novel graph wavelet packets construction, to our knowledge different from the ones known in the literature. We are inspired by the Spectral Graph Wavelet Transform defined by Hammond *et al.* in [1], which is based on a spectral graph wavelet at scale $s > 0$, centered on vertex n, and a spectral graph scaling function. Moreover, after defining the wavelet packet spaces and the associated tree, we obtain a dictionary of frames for $\mathbb{R}^N$ with known lower and upper bounds. We give concrete examples of how the wavelet packets can be used for compression, denoising, and reconstruction by considering a signal given by the fRMI data on the nodes of a voxel-wise brain graph G with 900,760 nodes (representing the brain voxels) defined in [2, 3].

References

[1] D. K. Hammond, P. Vandergheynst, and R. Gribonval, *Wavelets on graphs via spectral graph theory*, Appl. Comput. Harmon. Anal., 30(2):129–150, 2011.

[2] A. Tarun, D. Abramian, M. Larsson, H. Behjat, and D. Van De Ville, *Voxel-Wise Brain Graphs from Diffusion-Weighted MRI: Spectral Analysis and Application to Functional MRI*, preprint, 2021.

[3] A. Tarun, H. Behjat, T. Bolton, D. Abramian, and D. Van De Ville, *Structural mediation of human brain activity revealed by white-matter interpolation of fMRI*, NeuroImage, 213:116718, 2020.

[4] I. M. Bulai and S. Saliani, *Spectral graph wavelet packets frames*, Appl. Comput. Harmon. Anal., 66:18–45, 2023.

Data-intrinsic approximation in metric spaces

Jürgen Dölz (University of Bonn)

Co-authors:
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Abstract

Analysis and processing of data is a vital part of our modern society and requires vast amounts of computational resources. To reduce this computational burden, the compression and approximation of data has become of utmost importance. We consider the approximation of labeled data samples, mathematically described as site-to-value maps between finite metric spaces. In this setting, we identify the modulus of continuity as a valuable data-intrinsic tool to measure regularity of site-to-value maps without making additional assumptions. Based on this observation, we discuss consistency of this regularity measure in the infinite data limit and its efficient computational evaluation. Based on these results, we introduce a sample-based approximation theory for labeled data. For data subject to statistical uncertainty, we construct multilevel approximation spaces and a variant of the multilevel Monte Carlo method to compute statistical quantities of interest. Our considerations connect approximation theory for labeled data in metric spaces to the covering problem for (random) balls on the one hand, and the efficient evaluation of the modulus of continuity to combinatorial optimization on the other hand.

Graph Wedgelets: adaptive decomposition of graph signals based on wedge partitioning trees and geometric wavelets

Wolfgang Erb (University of Padua)

Abstract

In this presentation, we will briefly introduce graph wedgelets, an adaptive signal-processing tool for the decomposition and approximation of functions on graphs. Graph wedgelets approximate signals on graphs by piecewise constant or polynomial functions on adaptively generated binary wedge partitionings. They can be regarded as discrete variants of continuous wedgelets and binary space partitionings in the two-dimensional Euclidean space. The latter have, for instance, applications in the compression of images. We prove that continuous results on best m-term approximation with geometric wavelets can be transferred to the discrete graph setting, and show that the wedgelet representation of graph signals can be encoded and implemented in a simple way by a binary tree structure. We will also illustrate how this graph-based method can be applied for the compression and segmentation of images.

Kernel interpolation on generalized sparse grids

Helmut Harbrecht (University of Basel)

Co-authors:
Michael Griebel
Michael Multerer

Abstract

We consider scattered data approximation on product regions of equal and different dimensionality. On each of these regions, we assume quasi-uniform but unstructured data sites and construct optimal sparse grids for scattered data interpolation on the product region. For this, we derive new improved error estimates for the respective kernel interpolation error by invoking duality arguments. An efficient algorithm to solve the underlying linear system of equations is proposed. The algorithm is based on the sparse grid combination technique, where a sparse direct solver is used for the elementary anisotropic tensor product kernel interpolation problems. The application of the sparse direct solver is facilitated by applying a samplet matrix compression to each univariate kernel matrix, resulting in an essentially sparse representation of the latter. In this way, we obtain a method that is able to deal with large problems up to billions of interpolation points, especially in the case of reproducing kernels of nonlocal nature. Numerical results are presented to qualify and quantify the approach.

On the convergence of multiscale kernel regression under minimalistic assumptions

Armin Iske (University of Hamburg)

Abstract

We analyse the convergence of data regression in reproducing kernel Hilbert spaces (RKHS). This is done under minimalistic (i.e., mild as possible) assumptions on the data and on the kernel. To this end, we first prove convergence in the RKHS norm for just one fixed kernel. Our results are then transferred to a sequence of multiple scaled kernels, whereby we obtain convergence rates of multiscale kernel regression with respect to both the RKHS norm and the maximum norm. Supporting numerical results are finally presented.

Multiscale Geometric Methods for high-dimensional data near low-dimensional sets

Mauro Maggioni (Johns Hopkins University)

Abstract

We discuss a family of ideas, algorithms, and results for analyzing in a multiscale fashion data sets modeled by sampling from a probability measure in high dimensions that is concentrated near a low-dimensional set. They rely on the idea of performing suitable multiscale geometric decompositions of the data, and exploiting such decompositions to perform a variety of tasks in signal processing and statistical learning. In particular, we will discuss the problems of dictionary learning, of manifold learning, of regression on manifolds, of estimating probability measures on manifolds, and of efficiently computing optimal transportation plans between discrete probability measures.

Multilevel Preconditioning on Quantum Computers

Daniel Peterseim (University of Augsburg)

Co-authors:
Matthias Deiml

Abstract

This talk presents a quantum algorithm for efficiently solving linear systems that arise from the finite element discretization of elliptic partial differential equations. The key ingredient is a multilevel preconditioner of BPX type, which transforms the system into one with a uniformly bounded condition number. This makes it suitable for quantum linear system solvers. For any fixed spatial dimension, the algorithm computes linear functionals of the solution with rate-optimal complexity proportional to the inverse of the error tolerance, up to logarithmic terms. The method does not require smooth solutions and can handle rough coefficients, including random inputs. We demonstrate the feasibility of this approach through implementations on quantum circuit simulators and runs on current quantum hardware.

Superconvergence in Kernel Interpolation: A General Framework and Algorithmic Implications

Gabriele Santin (Ca' Foscari University of Venice)

Co-authors:
Toni Karvonen
Tizian Wenzel

Abstract

Kernel interpolation is a fundamental technique for approximating functions from scattered data, with a classical convergence theory when the target function lies in a reproducing kernel Hilbert space. In this talk, I will focus on the superconvergence regime, where the target function is smoother than the native space, and approximation errors decay faster than standard theory predicts. I will present a unified framework for superconvergence, characterizing it in terms of functions lying in ranges of specific operators, such as adjoints of embeddings. This perspective allows us to describe accelerated convergence across interpolation scales, and to link these phenomena to Mercer operators and associated power spaces. Applications to Sobolev spaces will be discussed in detail, showing how boundary conditions critically influence the effect. Finally, I will highlight implications for greedy and related kernel-based algorithms, where understanding superconvergence can guide both theoretical analysis and practical implementation.

On the Computation of Singular Integrals over Convex Polytopes

Johannes Tausch (Southern Methodist University)

Abstract

Singular integrals over convex polytopes arise in several applications, such as Galerkin discretizations of singular integral equations, Markov processes, and quantum mechanics. Sauter, Schwab, and co-workers have introduced a sequence of transformations that reduce singular integrals to integrals over hypercubes with smooth integrands. However, this construction is very much dependent on the type of polytope, and is therefore limited to Cartesian products of simplices and parallelotopes. This talk will approach singular integrals from the perspective of polytope theory. The centerpiece is a simple algorithm that decomposes any polytope into a collection of convex hulls of two lower-dimensional polytopes. The singularity on these pieces can then be handled effectively with Gauss-Jacobi quadrature rules. The method and its complexity are illustrated on Cartesian products of simplices and cubes in any dimension. We also give a geometric interpretation of the decompositions, which can be used to guide adaptivity for highly irregular polytopes.

Multiscale Approximation by Radial Basis Functions

Holger Wendland (University of Bayreuth)

Abstract

Radial basis functions (RBFs) represent a popular meshfree discretisation method. They are used in various areas comprising, for example, scattered data approximation, computer graphics, machine learning, aeroelasticity, and the geosciences. The approximation space is usually formed using the shifts of a fixed basis function. This simple approach makes it easy to construct approximation spaces of arbitrary smoothness and in arbitrary dimensions. Multiscale RBFs employ radial basis functions with compact support. In contrast to classical RBFs, they do not only use the shifts of a fixed basis function but also vary the support radius in an orderly fashion. If done correctly, this leads to an extremely versatile and efficient approximation method. In this talk, I will introduce the basic ideas of multiscale RBFs, I will give and analyse an explicit algorithm for the reconstruction of multivariate functions from scattered data. After that, I will discuss more recent results on the topic, for example, new representations of the underlying basis, how multiscale RBFs can be used for data compression, and for the resolution of different scales in the target function.

Multiscale greedy kernel algorithms

Tizian Wenzel (Ludwig Maximilian University of Munich)

Co-authors:
Sara Avesani
Michael Multerer
Armin Iske

Abstract

Both multiscale kernel approximation as well as greedy kernel algorithms are well-established and successful tools for scattered data approximation. In this talk, we first review recently established convergence analysis for greedy kernel approximation. Subsequently, we combine multiscale kernel approximation with greedy kernel algorithms and introduce multiscale greedy kernel algorithms. We show that these multiscale greedy algorithms can be understood as a generalization of standard multiscale approximation. Moreover, we provide a convergence analysis, leveraging enhanced convergence rates of target-data-dependent greedy algorithms. Benefits and challenges of these algorithms are discussed, and numerical results illustrate the new class of methods.

Participants

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