

# EQUILIBRIUM COUNTERFACTUALS: JOINT ESTIMATION AND CONTROL WITH STRUCTURAL MODELS\*

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## Abstract

The objective of applied structural microeconometrics is to identify policy-invariant parameters so alternative policies can be assessed. However, the common practice of treating policy changes as zero probability "counterfactuals" violates rational expectations: Agents understand policy changes are positive probability events which the structural estimation is intended to inform. We analytically characterize the implications for moment-based parameter inference. As shown, if a policy change is optimal, inference is biased. Further, the standard identifying assumption, constant partial derivative sign, is neither necessary nor sufficient with policy control. We offer an alternative identifying assumption: constant total differential sign with inference-policy feedback. It is shown that under this assumption, rational expectations can be imposed computationally (algorithmically) to generate unbiased inference and optimal policy. The quantitative importance of these effects in applied settings is illustrated by calibrating the Leland (1994) model to the Tax Cuts and Jobs Act of 2017.

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## 1. Introduction

In recent years, there have been intense debates regarding alternative microeconomic methodologies, particularly between advocates of reduced-form quasi-experimental methods and those advocating structural models. The natural experiment camp contends that the Achilles heel of structural work is an inability to deal with key issues concerning selection and heterogeneity. Conversely, the structural camp has argued that an important weakness of reduced form work is that it faces severe limitations on external validity. For examples, see Keane (2010) and Rust (2010).

Of course, limits on external validity give rise to challenges associated with using reduced-form methods to assess alternative “counterfactual” policies. In light of these challenges, the reduced-form school often focuses on so-called causal questions (“Is a particular causal mechanism operative?”) and avoids any formal treatment of normative questions. Or rather, reduced-form evidence is often left as “suggestive” of directions for future policy.

In contrast to reduced-form econometricians, the structural econometrician first writes down an explicit model that, ideally, mimics the real-world data generating process confronting agents, including an accounting for technologies, tastes, constraints, information sets, and shocks. The structural econometrician then selects empirical moments that are informative about unknown parameters and minimizes the distance between model-implied moments and the selected empirical moments. The notion of informativeness is formalized and analyzed by Gallant and Tauchen (1996), for example. Kahn and Whited (2017) provide the following heuristic description of moment selection: “From an intuitive perspective, this identification condition means that the relations between the moments and parameters need to be steep and monotonic...”

Advocates often argue that a key relative advantage of structural inference over reduced-form inference is that structural models allow for a rigorous assessment of alternative policy options that a government may adopt in the future. After all, the objective of structural inference is to estimate the magnitudes of policy-invariant (“deep”) parameters. With unbiased estimates of policy-invariant parameters in-hand, alternative policies can be fed into the model and welfare implications can be evaluated. This procedure is nicely summarized by Sargent (1984):

Once estimates of the free parameters of agent’s objectives and constraints have been obtained, the aim is to use them to analyze how the economy would behave under

hypothetical strategies for setting government policy variables that are different from the one evident in the sample period.

In this paper, we explore a logical contradiction that exists at the core of structural microeconomic methods when these methods are used in a manner consistent with their alleged advantage over reduced-form methods, as tools for assessing policy alternatives and for giving policy advice. To see the contradiction, consider that in specifying the decision problem of agents inside her model, the structural econometrician must specify government policy. Critically, it is customary to parameterize the structural model according to the status-quo government policy. But if the status quo government policy will remain in place, the econometric analysis is moot and the alleged advantage of structural methods in guiding policy is of no real value. Conversely, if “counterfactual” policy changes are positive probability events, agents with rational expectations will understand this, resulting in a change in their optimal actions.

This argument is in the spirit of Sargent (1987) who states, “There is a logical difficulty in using a rational expectations model to give advice, stemming from the self-referential aspect of the model that threatens to absorb the economic adviser into the model... That simultaneity is the source of the logical difficulties in using rational expectations models to give advice about government policy.” Striking a pessimistic note, Sargent (1984) stated, “I am unaware of an alternative approach to Sims or to rational expectations econometrics that avoids these contradictions and tensions.” These philosophical and logical difficulties apparently led Sargent to shy away from the use of macroeconomic models for the purpose of informing monetary and fiscal policy decisions. For example, Sargent (1994) states, “That’s a hard problem. I don’t make policy recommendations.”<sup>1</sup>

The pessimistic conclusion reached by Sargent (1994) was apparently not shared by all in the profession at the time. For example, Cooley, LeRoy and Raymon (1984) argued that, “Contrary to Lucas, there is no reason in principle why economists should decline to analyze specific historical episodes— that is, should be unwilling to rank different policy sequences evolving out of a common past (of course, this is not to minimize the practical difficulties attending such an exercise).”

Apparently, this set of philosophical debates has largely escaped the attention of structural microeconomicians. This state of affairs is not entirely surprising given that the debates between

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<sup>1</sup>Quoted in Sent (1998).

Sargent and Sims, amongst others, centered on the correct interpretation and utilization of vector autoregressions in the setting of monetary and fiscal policy.

With the preceding discussion in mind, the objectives of this paper are multi-fold. First, we develop a tractable general framework for incorporating agents' rational expectations into a structural microeconomic model that will be used for the purpose of policy advice. Second, and more importantly, we move well beyond highlighting philosophical contradictions by characterizing analytically the precise nature of bias that will arise if the structural econometrician engages in the customary practice of treating agents as being unaware of the potential for an endogenous policy change. Third, we discuss how, within the context of moment-based microeconomics, the philosophical challenges can be circumvented, while achieving unbiased parameter inference and optimal policy. Finally, we show that the standard moment monotonicity condition is neither necessary nor sufficient for parameter identification in the context of joint estimation and policy control, and we offer an alternative moment selection criterion.

In the model, there is a continuum of atomistic firms that will be observed by an econometrician. The firms are privately endowed with a single deep structural parameter, with knowledge of this parameter being sufficient for the government to set policy optimally. The econometrician will observe an empirical moment which will serve as the basis for her parameter inference. Importantly, under technical assumptions, the empirical moment is sufficient for the government to achieve unbiased parameter inference and the first-best policy.

When the model opens, the policy variable is initially equal to the pre-determined status quo value. Next, nature draws the unknown parameter  $u$  from the real line. The firms then privately observe this parameter and choose their preferred actions non-cooperatively. The econometrician then observes a moment  $m$  arising from the endogenous decisions of the firms. Importantly, the moment is assumed to satisfy the *standard monotonicity condition* (constant sign of  $\partial m/\partial u$ ). The econometrician then matches her model moment with the observed empirical moment in order to draw an inference  $\hat{u}$  of the unknown parameter  $u$ . Finally, with some positive probability the government will subsequently enjoy an opportunity to re-set its policy variable based upon the econometric inference.

We begin the analysis by characterizing the bias that arises if the structural microeconomician treats policy changes as counterfactual events and thus fails to impose the assumption that agents

have rational expectations regarding future policy. Indeed, within the model, firms know the true value of the parameter  $u$ . Based upon this information, firms can correctly anticipate the econometrician's inference  $\hat{u}(u)$ . More generally, firms can rationally anticipate the function  $\hat{u}(\cdot)$  which determines the econometrician's inference for each possible realized value  $u$ . Armed with this knowledge, the firms then correctly anticipate what policy the government will implement if it enjoys policy discretion.

As shown, the failure to impose rational expectations results in  $\hat{u}(u) \neq u$  at all points except the one possible realization of the unknown parameter, we call it  $u_0$ , that would justify the government maintaining the status quo. Intuitively, in this exceptional (measure zero) case, the econometrician will not violate rational expectations since parameterizing the model as if the status quo will remain in place in the future is actually correct here. At all other points  $u \in \mathbb{R}$ , inference is biased.

We next address head-on the “practical difficulties” alluded to by Cooley, LeRoy and Raymon (1984), doing so in the context of applied structural microeconometrics. This allows us to offer a simple catalog of the nature of bias as a function of the properties of the moment function  $m$  and the government policy function  $g$ . The central insight is as follows. In a rational expectations equilibrium, the empirical moment first varies directly with changes in the parameter  $u$ . This direct effect,  $m_u$ , is accounted for by the structural econometrician. However, the empirical moment also varies indirectly through feedback from the inference ( $\hat{u}$ ) to discretionary government policy ( $g(\hat{u})$ ) to the empirical moment. Thus we write the empirical moment as  $m[u, g(\hat{u}(u))]$ . It is this *indirect effect* arising from policy control,  $m_g g' \hat{u}'$ , that is generally ignored.

The nature of bias depends on the nature of the indirect effect. There are three cases to consider. In the first case, the sign of the indirect effect is opposite to that of the direct effect. Here, the estimated parameter overshoots for  $u < u_0$  and then undershoots for  $u > u_0$  (and recall,  $u_0$  justifies the status quo). Intuitively, the modeler incorrectly treats small changes in the empirical moment to small changes in  $u$  because she here fails to account for the countervailing indirect effect. In the second case, the indirect effect is small in absolute value and has the same sign as the direct effect. Here, the estimated parameter undershoots for  $u < u_0$  and overshoots for  $u > u_0$ . Intuitively, the modeler incorrectly treats large changes in the empirical moment to large changes in  $u$  because she here fails to account for the amplifying indirect effect. In the third case, the indirect effect is large in absolute value and has the same sign as the direct effect. Here, the estimated parameter actually

decreases with the true parameter, and it is actually possible for an equilibrium to arise where the estimated parameter always has the wrong sign. The subtle intuition for this case is provided in the body of the paper.

We illustrate the quantitative significance of these effects by considering an econometrician whose objective is to infer bankruptcy costs using the canonical structural model of Leland (1994). In particular, we consider the recent cut in the corporate income tax rate implemented under the Tax Cuts and Jobs Act of 2017. Here the structural microeconomist backs out implied bankruptcy costs from observed values of corporate interest coverage ratios. By assumption, the econometrician knows the underlying real technology but fails to impose the assumption of rational expectations on the part of firms. In our calibrated example, this leads to an eight-fold overstatement of bankruptcy costs. Intuitively, firms rationally anticipate a tax cut and thus choose low leverage in light of the low value of future debt tax shields. Neglecting this fact, the econometrician mistakenly infers that the low leverage stems from extremely high bankruptcy costs.

Importantly, we show that, at least in terms of structural microeconomics, the paradoxes discussed by Sargent (1984, 1987) are not insurmountable. In particular, we describe how unbiased parameter inference and optimal government policy can be achieved. Essentially, the econometrician must insist that the beliefs of the agents inside her model are consistent with the policy advice she will give. This involves recognizing that the observed moment will vary directly with the unknown parameter  $u$  and also with agents' rational expectation of government policy changes in light of (correct) inference of the structural parameter. Related to this point, the standard moment monotonicity condition, which focuses on partial derivatives of moments, is neither necessary nor sufficient for parameter identification, and we propose total derivatives, cum policy feedback, as the correct moment selection criterion in the context of joint estimation and control exercises.

It is worth emphasizing at the outset that our focus below on structural parameter inference for the purpose of government policy setting towards firms is simply to fix ideas. In broad details, the model applies to any entity attempting to infer some parameter for a group of agents, provided that the agents are forward-looking rational maximizers, and provided that the entity will act based upon its parameter inference with positive probability. For example, firms might be interested in estimating policy-invariant consumer parameters with the goal of evaluating alternative pricing

strategies, product characteristics, or marketing expenditures.<sup>2</sup> Alternatively, governments might be interested in estimating deep parameters determining labor supply, human capital investments, or the number of offspring.<sup>3</sup>

It is also worth emphasizing at the outset that our model is intended to be forward-looking in terms of its application. That is, it might be plausibly argued that in many settings the link between econometric evidence and decision-making is weak at the present time. However, most economists strongly advocate a move toward systematic evidence-based decision-making by governments and firms. Moreover, it is clear that governments notwithstanding, modern firms are increasingly moving toward data-driven decision-making, or fail to do so at their own peril. Our framework can be seen as capturing the types of problems of econometric inference that will intensify as systematic data-driven decision-making becomes commonplace, and as the measured agents understand this to be taking place. The specific focus of this paper is on potential bias in structural parameter inference.<sup>4</sup>

The rest of the paper is as follows. Section 2 describes the economic setting. Section 3 characterizes the nature of bias if the econometrician fails to impose rational expectations. Section 4 shows how, under technical conditions, unbiased parameter inference and first-best government policy can be achieved through a fully-consistent application of rational expectations. In addition, Section 4 shows how traditional moment selection criteria are altered when one moves away from a pure estimation setting to a setting with joint estimation and control. Section 5 presents a quantitative example. Section 6 considers a multivariate extension.

## **2. The Economic Setting**

We consider a univariate parameter inference problem where the econometric model is exactly identified. The first subsection describes timing and technology assumptions. The second subsection illustrates how the general framework maps to a specific applied microeconomic problem.

### **2.1. Timing, Technology, and Beliefs**

There is a representative sample consisting of a continuum of atomistic firms privately endowed with a policy-invariant (“deep”) structural parameter. Knowledge of this parameter is sufficient for the government to set policy optimally.

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<sup>2</sup>See Chintagunta, Erdem, Rossi and Wedel (2006) for a survey of structural methods in marketing.

<sup>3</sup>See Wolpin (2013).

<sup>4</sup>Chemla and Hennessy (2018) analyze natural experiments.

An econometrician will observe an empirical moment derived from the measured actions of the sample firms. To fix ideas, one can think of the moment as being the sample mean of investment, new employees, R&D, or leverage. In practice, moments such as variance, skewness, or kurtosis may also be informative about deep parameters. In the context of indirect inference, the moment can be the coefficient obtained when firm decision variables are regressed on observable covariates, such as the coefficient on market-to-book ( $Q$ ) in an investment regression. Below, specific examples are provided.

The econometrician has developed a structural model and will match her model-implied moment with the observed empirical moment. Importantly, under conditions derived below, if the econometrician were to impose rational expectations in a fully internally consistent manner, this moment matching procedure would allow her to infer the true value of the deep parameter and the government would then be able to correctly determine the optimal policy.

The atomistic firms are rational, forward-looking, and act non-cooperatively. Each atomistic firm correctly understands it cannot change the moment observed by the econometrician by unilaterally changing its own action.

The deep parameter, denoted  $u$ , is common to all sample firms. However, this assumption does not preclude firm heterogeneity. For example, firms may be identical ex ante but face idiosyncratic shocks ex post. Alternatively, firms may face idiosyncratic shocks that alter their measured actions. Finally, firm-level parameters might be, say, multiples of a common “aggregate” parameter  $u$ , e.g.  $u_i = \varepsilon_i u$  where  $\varepsilon_i$  is a firm-specific scalar known by firm  $i$ . An alternative technological assumption, not adopted here, is that each firm receives a noisy signal of the common parameter  $u$ . In such a setting, as in the present setting, parameter inference would need to account for feedback from inference to the policy variable.

The parameter  $u$  represents the realization of a random variable  $\tilde{u}$  with cumulative distribution function  $\Psi$  with a strictly positive density  $\psi$  on  $\mathbb{R}$  with no atoms. The realized parameter  $u$  is privately observed by each of the sample firms, but unobservable to the econometrician and the government. Below,  $\hat{u}(u)$  denotes an equilibrium parameter inference by the econometrician in the event that  $\tilde{u} = u$ , with  $\hat{u}(\cdot)$  denoting an equilibrium inference function.

Timing is as follows. When the model opens at time  $t = 0$ , the government policy variable is initially equal to the pre-determined *status-quo*  $\gamma_0 \in \Gamma$  where the set of feasible government

policies  $\Gamma \equiv (\underline{\gamma}, \bar{\gamma})$ . Next, nature draws  $u$  according to the distribution function  $\Psi$ . Each sample firm  $i$  then chooses an optimal *pre-inference action*  $\phi_i$ . This action can be multi-dimensional. The econometrician then observes the *empirical moment*  $m$ , which is derived from the pre-inference actions of the sample firms. Next, the econometrician will attempt to match her model-implied moment with the empirical moment, resulting in *parameter inference*  $\hat{u}$ . The econometrician then reports  $\hat{u}$  to the government. All of these events take place at the initial time  $t = 0$ .

Time is either discrete or continuous and the horizon can be finite or infinite. There is an independent stochastic process  $d$  such that for all  $t \geq 0$ ,  $d_t \in \{0, 1\}$ . Let

$$t^* \equiv \inf_{t \geq 0} d_t = 1.$$

At time  $t^*$ , the government enjoys a one-time opportunity to permanently re-set the policy variable, having already received the econometrician's report. At all prior dates, policy is fixed at the status quo  $\gamma_0$ .

The government's equilibrium choice of *discretionary policy* is denoted  $\gamma^*$ . Under the stated assumptions, government policy post-inference is an independent stochastic process  $\tilde{\gamma}_t$  with

$$\begin{aligned} t < t^* &\Rightarrow \tilde{\gamma}_t = \gamma_0 \\ t \geq t^* &\Rightarrow \tilde{\gamma}_t = \gamma^*. \end{aligned} \tag{1}$$

No sample firm receives any signal that is informative about  $\gamma^*$  aside from  $u$ . Thus, firm policy expectations are homogeneous. With this in mind, let  $\gamma$  denote the value of  $\gamma^*$  anticipated by the sample firms conditional upon their knowledge of  $u$ .

The optimal pre-inference action of firm  $i$  can be expressed as

$$\phi_i(u, \gamma; \gamma_0) \tag{2}$$

where the subscript  $i$  captures idiosyncratic shocks and the semi-colon separates variables from the constant  $\gamma_0$ .

It is assumed that observation of a continuum of sample firms is sufficient to ensure that any idiosyncratic shocks have no effect on the observed moment, so that  $m$  can be expressed as  $m(u, \gamma; \gamma_0)$ . For brevity, the constant  $\gamma_0$  will be suppressed and the empirical moment will be

represented by the following mapping:

$$m : \mathbb{R} \times \Gamma \rightarrow \mathbb{R}. \tag{3}$$

The first argument in the moment function  $m$  is the unknown parameter  $u \in \mathbb{R}$ . The second argument in the moment function is anticipated discretionary government policy  $\gamma \in \Gamma$ .

The following assumption ensures the setting considered is seemingly-ideal.

**Assumption 1.** *The model-implied moment function is identical to the empirical moment function  $m : \mathbb{R} \times \Gamma \rightarrow \mathbb{R}$ . Moreover, for each  $\gamma \in \Gamma$ , the function  $m(\cdot, \gamma)$  is continuously differentiable and strictly monotonic.*

The first part of Assumption 1 states that the structural model is correct. In particular, from Assumption 1 it follows that if the model were to be parameterized with a correct stipulation of  $u$  and  $\gamma$ , the model-implied moment would match the empirical moment. The second part of Assumption 1 is the traditional structural identifying assumption that  $m(\cdot, \gamma)$  is strictly monotonic.

We next characterize how the moment varies with anticipated discretionary government policy.

**Assumption 2.** *For each  $u \in \mathbb{R}$ ,  $m(u, \cdot)$  is a continuously differentiable strictly monotonic function.*

Notice, the setting considered is quite general. For example, as in Blume, Easley and O’Hara (1982), one can think of the sample firms as solving canonical finite or infinite horizon dynamic programming problems with differentiable policy functions where monotone comparative statics apply and carry over to  $m$ . Nevertheless, it is worth emphasizing that in order for Assumption 2 to hold, it must be the case that the sample firms are solving forward-looking problems in which anticipated discretionary government policy  $\gamma$  enters as a relevant parameter in their program, either through periodic payoff functions, constraint functions, and/or transition functions.

The function  $g : \mathbb{R} \rightarrow \Gamma$  represents optimal discretionary government policy. If the government had the ability to directly observe  $u$ , its optimal discretionary policy would be  $g(u)$ . Of course, the sample firms will have already chosen their pre-inference actions  $\phi_i$ . However, the government correctly understands that should it enjoy discretion, its policy choice  $\gamma^*$ , in addition to the parameter  $u$ , will determine the post-inference actions of the sample firms and/or other agents in the economy, e.g. future generations of firms. The function  $g$  represents the socially optimal  $u$ -contingent government policy in light of the relevant tradeoffs. The following assumption is imposed.

**Assumption 3.** *The optimal government policy  $g$  is a continuously differentiable strictly monotonic function mapping  $\mathbb{R}$  onto  $\Gamma$ .*

The government is presumed to believe that the standard moment matching exercise will allow the econometrician to deliver a correct estimate of the unknown parameter. Critically, Assumption 1 would seem to imply that this confidence is justified. After all, the model moment function is equal to the empirical moment function, and the moment is monotone in the unknown parameter.<sup>5</sup> We have the following assumption.

**Assumption 4.** *The government chooses discretionary policy optimally given its belief that for all  $u \in \mathbb{R}$ ,  $\hat{u}(u) = u$ .*

From Assumption 4 it follows that for all  $u \in \mathbb{R}$ , the *endogenous discretionary policy* of the government is

$$\gamma^*(u) = g[\hat{u}(u)]. \quad (4)$$

An alternative interpretation of condition (4) is that the function  $g$  represents equilibrium policy outcomes from an extensive form game in which the econometrician's parameter estimate is fed into the political decision-making process. This alternative interpretation would not alter the characterization of bias below, but would necessarily rule out characterization of the welfare consequences of biased parameter inference.

We posit that *real-world firms* form rational expectations. In particular, real-world firms know that the government may enjoy policy discretion at some future date. They also know the government will place full faith in the econometrician's structural parameter estimate  $\hat{u}$ , and will then input this estimate into the policy function  $g$ . The following assumption formalizes this specification of firm beliefs.

**Assumption 5 [Firm Rational Expectations].** *For all  $u \in \mathbb{R}$ , real-world firms correctly anticipate discretionary government policy, with*

$$\gamma(u) = \gamma^*(u) = g[\hat{u}(u)]. \quad (5)$$

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<sup>5</sup>See Gallant and Tauchen (1996) and Kahn and Whited (2017) for moment monotonicity as an identifying assumption.

The first equality in the preceding equation ensures that  $\gamma(\cdot)$  satisfies rational expectations. The second equality reflects how discretionary government policy  $\gamma^*$  will actually be formed in equilibrium, with  $\hat{u}(u)$  being fed into  $g$ . Effectively, under rational expectations, the real-world firms infer the econometrician's parameter estimate which allows them to correctly anticipate discretionary government policy.

From the preceding equation it follows that the empirical moment observed by the econometrician is:

$$m[u, \gamma(u)] = m[u, \gamma^*(u)] = m[u, g(\hat{u}(u))]. \quad (6)$$

In reality, the post-inference government policy follows the stochastic process described in equation (1). The real-world sample firms have rational expectations and understand this. However, we assume the econometrician departs from rational expectations by parameterizing her structural model according to the status-quo. Below we formally state this important assumption.

**Assumption 6 [Status Quo Parameterization].** *Firms inside the structural model anticipate that the status quo will be maintained even if the government enjoys policy discretion, believing  $\gamma = \gamma_0$ .*

Notice, by parameterizing her model according to the status quo, the econometrician implicitly treats the firms as being unaware of her own activities and the policy function they are intended to serve, informing the government's discretionary decisions. Below we analyze the implications for parameter inference and government policy.

From the preceding discussion it follows that for all  $u \in \mathbb{R}$ , the structural econometrician's parameter estimate will be derived from the following *inference equation*

$$m[u, \gamma^*(u)] = m[\hat{u}(u), \gamma_0] \quad (7)$$

or

$$m[u, g(\hat{u}(u))] = m[\hat{u}(u), \gamma_0]. \quad (8)$$

The left side of the preceding equation is the observed empirical moment. The empirical moment reflects the fact that the sample firms will choose their pre-inference actions optimally given the true parameter value  $u$  and their correct anticipation of discretionary government policy (Assumption 5). The right side of the preceding equation is the model-implied moment under the status quo

parameterization (Assumption 6). The estimated parameter  $\hat{u}(u)$  is chosen so that the model implied moment is equal to the observed empirical moment.

## 2.2. Example: Inferring Labor Adjustment Costs

Consider an econometrician who wants to estimate a labor adjustment cost parameter  $u$  based upon some empirical moment, say, the average gross increase in firm or plant-level employment.<sup>6</sup> In particular, the econometrician would like to infer the marginal cost to firms of complying with government employment regulations. For simplicity, assume that just after receiving the econometrician's report, the government will enjoy policy discretion with probability  $p > 0$ .

This exercise is in the spirit of Hammermesh (1989), Blanchard and Portugal (2001), and Ejarque and Portugal (2007) who estimate parameters of labor adjustment cost functions and then use the estimates as the basis for making policy recommendations regarding labor market reforms. Of course, although the focus of the example is on labor adjustment costs, similar arguments would apply to moment-based inference of capital stock adjustment cost parameters. See Adda and Cooper (2003) for an overview.

Let  $\phi_i$  denote the number of workers hired by firm  $i$ . Prior to inference by the econometrician, each sample firm  $i$  is assumed to solve the following program featuring a standard quadratic technology:

$$\max_{\phi_i} \quad \phi_i q - \frac{1}{2}[p\gamma + (1-p)\gamma_0]N(u)(\phi - \varepsilon_i)^2. \quad (9)$$

In the preceding equation,  $q > 0$  represents the shadow value of a hired worker—the expected present value of worker marginal product less wages. For simplicity, assume  $q$  is known to the econometrician.<sup>7</sup> Marginal costs of complying with labor market regulations is captured by the product of expected units of regulation and the function  $N$  measuring the marginal cost of complying with each unit of regulation, again expressed in present value terms. The function  $N$  is, say, the normal cumulative distribution function. Finally,  $\varepsilon_i$  captures a mean-zero firm-specific shock to labor costs. In this way, the structural estimation allows for firm heterogeneity.

The sample firms will solve the preceding program, and the econometrician will observe the

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<sup>6</sup>We focus here on hiring costs since firing costs may well differ.

<sup>7</sup>Or the econometrician is willing to rely upon existing estimates of this parameter.

following empirical moment:

$$\int_i \phi_i di = m[u, \gamma^*(u)] = [pg(\hat{u}(u)) + (1-p)\gamma_0]^{-1}[N(u)]^{-1}q. \quad (10)$$

Evaluated at parameterization  $u'$ , the model implied moment is:

$$m(u', \gamma_0) = [\gamma_0]^{-1}[N(u')]^{-1}q. \quad (11)$$

The econometrician chooses her parameter estimate so that empirical moment is just equal to the model-implied moment. It follows that in the present context the inference equation (8) is:

$$[pg(\hat{u}(u)) + (1-p)\gamma_0]^{-1}[N(u)]^{-1}q = [\gamma_0]^{-1}[N(\hat{u}(u))]^{-1}q. \quad (12)$$

Rearranging terms in the preceding equation we find

$$\hat{u}(u) = N^{-1} \left[ \frac{pg(\hat{u}(u)) + (1-p)\gamma_0}{\gamma_0} \times N(u) \right]. \quad (13)$$

From which it follows that

$$\hat{u}(u) = u \Leftrightarrow g(u) = \gamma_0. \quad (14)$$

That is, parameter inference is unbiased at point  $u$  if and only if the status quo is truly optimal at that point. Of course, such  $u$  occurs with probability zero, a distressing finding.

The next subsection offers a more general and precise characterization of the bias.

### 3. Bias Characterization

This section characterizes the nature of parameter inference and associated policy outcomes if the structural model fails to impose the assumption that firms have rational expectations.

Before proceeding, it will be convenient to express the differential form of the inference equation. In particular, under technical conditions derived below, there will exist a continuously differentiable function  $\hat{u}(\cdot)$  satisfying the inference equation (8). Assuming such a function exists, we have the following differential form:

$$m_u[u, g(\hat{u}(u))] + m_\gamma[u, g(\hat{u}(u))]g'[\hat{u}(u)]\hat{u}'(u) = m_u[\hat{u}(u), \gamma_0]\hat{u}'(u). \quad (15)$$

The differential form of the inference equation makes clear the potential for bias. The right side captures the econometrician's faulty inference procedure which is predicated upon the incorrect

assumption that firms expect the status quo to be maintained with probability 1. Thus, she incorrectly imputes any change in the observed moment to the direct effect as captured by the partial derivative,  $m_u$ . The left side of the preceding equation captures the true total differential of the empirical moment with respect to  $u$ . If  $u$  is perturbed, there will be direct effect on the moment as captured by the first term,  $m_u$ . In addition, the empirical moment will vary further due to the rational anticipation of firms that government policy will change based upon changes in the econometrician's parameter inference. This inference-policy feedback effect is captured by the second term on the left side of the equation ( $m_\gamma g' \hat{u}'$ ).

Let  $u_0$  be the unique value of the parameter  $u$  at which a fully-informed government would find it optimal to implement the status quo policy  $\gamma_0$ . That is

$$u_0 \equiv g^{-1}(\gamma_0) \Leftrightarrow g(u_0) = \gamma_0. \quad (16)$$

Uniqueness of  $u_0$  and invertibility follow from  $g$  being strictly monotone (Assumption 3).

The next proposition characterizes the realization(s) of the random variable  $\tilde{u}$  at which parameter inference will be unbiased.

**Proposition 1.** *Let the structural model be parameterized assuming government will implement  $\gamma_0$  (the status quo) when it enjoys policy discretion. Parameter inference is unbiased at point  $u$  if and only if  $g(u) = \gamma_0$ . There is a unique point at which this occurs,  $u_0 \equiv g^{-1}(\gamma_0)$ .*

**Proof.** Referring to the inference equation (7), it follows from the strict monotonicity of  $m$  in its first argument that

$$\gamma^*(u) = \gamma_0 \Rightarrow \hat{u}(u) = u.$$

Again referring to the inference equation (7), it follows from the strict monotonicity of  $m$  in its second argument that

$$\hat{u}(u) = u \Rightarrow \gamma^*(u) = \gamma_0.$$

Finally if point  $u$  is a point such that parameter inference is unbiased and the status quo is optimal then it must be that

$$\gamma^*(u) = g(u) = \gamma_0.$$

From the strict monotonicity of  $g$  the unique point at which this occurs,  $u_0$ . ■

The intuition for the preceding result is as follows. At any realization of  $u$  other than  $u_0$ , real-world firms anticipate the government will implement a policy different from the status quo should it enjoy policy discretion. The real-world firms then change their optimal behavior accordingly, leading to changes in the observed moment. However, under Assumption 6, the econometrician fails to take the inference-policy feedback effect into account, leading to bias.

Having established parameter inference will only be unbiased at point  $u_0$ , the next proposition provides insight into the nature of bias at all other  $u \in \mathbb{R}$ .

**Proposition 2.** *Let the inference equation (7) be satisfied at point  $u$  by  $\hat{u}(u)$ . If  $m_u m_\gamma > 0$ , then*

$$\begin{aligned}\gamma^*(u) < \gamma_0 &\Rightarrow \hat{u}(u) < u \\ \gamma^*(u) > \gamma_0 &\Rightarrow \hat{u}(u) > u.\end{aligned}$$

*If  $m_u m_\gamma < 0$ , then*

$$\begin{aligned}\gamma^*(u) < \gamma_0 &\Rightarrow \hat{u}(u) > u \\ \gamma^*(u) > \gamma_0 &\Rightarrow \hat{u}(u) < u.\end{aligned}$$

**Proof.** There are four cases to consider. Suppose first  $m$  is increasing in both arguments. Then from the inference equation (7) it follows

$$\begin{aligned}\gamma^*(u) < \gamma_0 &\Rightarrow m[u, \gamma_0] > m[u, \gamma^*(u)] \equiv m[\hat{u}(u), \gamma_0] \Rightarrow \hat{u}(u) < u \\ \gamma^*(u) > \gamma_0 &\Rightarrow m[u, \gamma_0] < m[u, \gamma^*(u)] \equiv m[\hat{u}(u), \gamma_0] \Rightarrow \hat{u}(u) > u.\end{aligned}$$

Suppose next  $m$  is decreasing in both arguments. Then

$$\begin{aligned}\gamma^*(u) < \gamma_0 &\Rightarrow m[u, \gamma_0] < m[u, \gamma^*(u)] \equiv m[\hat{u}(u), \gamma_0] \Rightarrow \hat{u}(u) < u \\ \gamma^*(u) > \gamma_0 &\Rightarrow m[u, \gamma_0] > m[u, \gamma^*(u)] \equiv m[\hat{u}(u), \gamma_0] \Rightarrow \hat{u}(u) > u.\end{aligned}$$

Suppose next  $m$  is decreasing in its first argument and increasing in its second argument. Then

$$\begin{aligned}\gamma^*(u) < \gamma_0 &\Rightarrow m[u, \gamma_0] > m[u, \gamma^*(u)] \equiv m[\hat{u}(u), \gamma_0] \Rightarrow \hat{u}(u) > u \\ \gamma^*(u) > \gamma_0 &\Rightarrow m[u, \gamma_0] < m[u, \gamma^*(u)] \equiv m[\hat{u}(u), \gamma_0] \Rightarrow \hat{u}(u) < u.\end{aligned}$$

Suppose finally  $m$  is increasing in its first argument and decreasing in its second argument. Then

$$\begin{aligned}\gamma^*(u) < \gamma_0 &\Rightarrow m[u, \gamma_0] < m[u, \gamma^*(u)] \equiv m[\hat{u}(u), \gamma_0] \Rightarrow \hat{u}(u) > u \\ \gamma^*(u) > \gamma_0 &\Rightarrow m[u, \gamma_0] > m[u, \gamma^*(u)] \equiv m[\hat{u}(u), \gamma_0] \Rightarrow \hat{u}(u) < u. \blacksquare\end{aligned}$$

The intuition behind the preceding result is as follows. Per Assumption 6, the econometrician's structural model incorrectly stipulates firm beliefs at any  $u$  at which the discretionary government policy will differ from the status quo. This incorrect stipulation of beliefs leads to incorrect inference. For example, taking the first part of the proposition, suppose the empirical moment function  $m$  is increasing (decreasing) in both arguments. Then if, say,  $\gamma^*(u) > \gamma_0$ , the moment will be higher (lower) than would be inferred based upon the direct effect  $m_u$ , causing  $\hat{u}$  to overshoot  $u$ . Taking the second part of the proposition, suppose that  $m_u > 0$  and  $m_\gamma < 0$ . Then if, say,  $\gamma^*(u) > \gamma_0$ , the moment will be lower than would be inferred based upon the direct effect  $m_u$ , causing  $\hat{u}$  to undershoot  $u$ .

The preceding proposition characterizes  $\hat{u}$  at a *particular point*  $u$  where the inference equation (7) has a solution. However, as shown below, the inference equation need not have a solution. With this in mind, the following lemma offers a sufficient condition such that there exists a (continuously differentiable) function  $\hat{u}(\cdot)$  that satisfies the inference equation pointwise for all  $u \in \mathbb{R}$ .

**Lemma 1.** *Let  $m_u m_\gamma < 0$  and  $g' > 0$  or let  $m_u m_\gamma > 0$  and  $g' < 0$ . Then there exists a continuously differentiable strictly monotonic increasing function  $\hat{u}(\cdot)$  satisfying the inference equation (7) for all  $u \in \mathbb{R}$ . The function  $\hat{u}(\cdot)$  has slope in  $(0, 1)$  at  $u_0$ .*

**Proof.** Consider the following function which is continuously differentiable in its two arguments

$$F(u, z) \equiv m[u, g(z)] - m(z, \gamma_0).$$

Any root  $z$  of the preceding equation represents a solution to the inference equation (7). We know (Proposition 1) the root at  $u_0$  is  $u_0$ . Consider next arbitrary  $u \neq u_0$ . Under the stated conditions it is readily verified that

$$\begin{aligned}F(u, u) &\equiv m[u, g(u)] - m(u, \gamma_0) \\ F(u, u_0) &\equiv m(u, \gamma_0) - m(u_0, \gamma_0)\end{aligned}$$

have opposite signs. From the Location of Roots Theorem, there exists a point  $\hat{u}$  solving the inference equation

$$F(u, \hat{u}) = 0.$$

Moreover, under the stated conditions

$$\frac{\partial}{\partial \hat{u}} F(u, \hat{u}) = m_\gamma[u, g(\hat{u})]g'(\hat{u}) - m_u(\hat{u}, \gamma_0) \neq 0.$$

It thus follows from the Implicit Function Theorem that there exists a continuously differentiable function  $\hat{u}(\cdot)$  defined on an interval  $I$  about the (arbitrary) point  $u$  such that

$$F[\tilde{u}, \hat{u}(\tilde{u})] = 0 \quad \forall \quad \tilde{u} \in I \tag{17}$$

and

$$\begin{aligned} \hat{u}'(u) &= \frac{m_u[u, g(\hat{u}(u))]}{m_u[\hat{u}(u), \gamma_0] - m_\gamma[u, g(\hat{u}(u))]g'[\hat{u}(u)]} \\ &= \left[ \frac{m_u[\hat{u}(u), \gamma_0]}{m_u[u, g(\hat{u}(u))]} - \frac{m_\gamma[u, g(\hat{u}(u))]g'[\hat{u}(u)]}{m_u[u, g(\hat{u}(u))]} \right]^{-1}. \end{aligned} \tag{18}$$

Notice, under the stated conditions, the term in square brackets in the preceding equation is strictly positive, implying the derivative of the function  $\hat{u}$  is positive. Finally, the last statement in the lemma follows from

$$\begin{aligned} \hat{u}'(u_0) &= \frac{m_u[u_0, g(\hat{u}(u_0))]}{m_u[\hat{u}(u_0), \gamma_0] - m_\gamma[u_0, g(\hat{u}(u_0))]g'[\hat{u}(u_0)]} \\ &= \frac{m_u[u_0, g(u_0)]}{m_u(u_0, \gamma_0) - m_\gamma[u_0, g(u_0)]g'(u_0)} \\ &= \left[ 1 - \frac{m_\gamma[u_0, g(u_0)]g'(u_0)}{m_u(u_0, \gamma_0)} \right]^{-1}. \blacksquare \end{aligned} \tag{19}$$

To illustrate the preceding lemma, and many that follow, it will be useful to define a *linear technology*:

$$\begin{aligned} m(u, \gamma) &\equiv \alpha u + \beta \gamma \\ g(u) &\equiv \kappa u \end{aligned} \tag{20}$$

where  $\alpha$ ,  $\beta$  and  $\kappa$  are arbitrary nonzero constants. Under the linear technology, the inference equation (8) can be written as

$$u + \kappa \hat{u}(u) = \hat{u}(u) + \gamma_0.$$

From equation (16) it follows that here  $\gamma_0 = \kappa u_0$ . Using this fact, and rearranging terms in the preceding equation, the inference equation can be expressed as

$$\alpha u - \beta \kappa u_0 = (\alpha - \beta \kappa) \hat{u}(u). \quad (21)$$

If  $\alpha = \beta \kappa$ , the preceding equation does not have a solution at any point other than  $u_0$ . Under the conditions in Lemma 1,  $\alpha \neq \beta \kappa$ . In fact, under the conditions specified in the lemma,  $\alpha$  and  $\beta \kappa$  have different signs. With  $\alpha \neq \beta \kappa$ , the solution to the linear technology inference equation is

$$\begin{aligned} \hat{u}(u) &= \frac{\alpha u - \beta \kappa u_0}{\alpha - \beta \kappa} = u + \frac{\beta \kappa (u - u_0)}{\alpha - \beta \kappa}. \\ \Rightarrow \hat{u}'(u) &= \frac{\alpha}{\alpha - \beta \kappa}. \end{aligned} \quad (22)$$

Under the conditions in Lemma 1,  $\hat{u}'$  is some constant in  $(0, 1)$ .

Lemma 1 leads directly to the following proposition.

**Proposition 3.** *Let  $m_u m_\gamma < 0$  and  $g' > 0$  or let  $m_u m_\gamma > 0$  and  $g' < 0$ . Then there exists a continuously differentiable strictly monotonic increasing function  $\hat{u}(\cdot)$  satisfying the inference equation. For all  $u < u_0$ ,  $\hat{u}(u) \in (u, u_0)$  and for all  $u > u_0$ ,  $\hat{u}(u) \in (u_0, u)$ . If  $g$  is increasing, then  $u < u_0$  implies  $\gamma^*(u) \in (g(u), \gamma_0)$  and  $u > u_0$  implies  $\gamma^*(u) \in (\gamma_0, g(u))$ . If  $g$  is decreasing, then  $u < u_0$  implies  $\gamma^*(u) \in (\gamma_0, g(u))$  and  $u > u_0$  implies  $\gamma^*(u) \in (g(u), \gamma_0)$ .*

**Proof.** The first statement in the Proposition is from Lemma 1. Next note that  $\hat{u}'(u_0) \in (0, 1)$ . It follows that for  $u$  on the left neighborhood of  $u_0$ ,  $\hat{u}(u) \in (u, u_0)$  and for  $u$  on the right neighborhood of  $u_0$ ,  $\hat{u}(u) \in (u_0, u)$ . From the continuity of  $\hat{u}(\cdot)$  and Proposition 1 it follows that for all  $u < u_0$ ,  $\hat{u}(u) > u$  and for all  $u > u_0$ ,  $\hat{u}(u) < u$ . From the strict monotonicity of  $\hat{u}(\cdot)$  it follows that for all  $u < u_0$ ,  $\hat{u}(u) < u_0$  and for all  $u > u_0$ ,  $\hat{u}(u) > u_0$ . The final two clauses follow from the fact that  $\gamma^* = g(\hat{u})$ . ■

Inspection of equation (15) reveals the intuition for the preceding proposition. Under the stated assumptions, the second term on the left side of the differential form of the inference equation (15) dampens the sensitivity of the moment to changes in  $u$ —an effect ignored by the econometrician. She will then incorrectly impute the small changes in the moment to small changes in  $u$ . That is,  $\hat{u}$  will tend to have a slope less than unity, with  $\hat{u}$  overshooting for  $u < u_0$  and undershooting for  $u > u_0$ .

These effects are illustrated in Figures 1, 2 and 3 which consider the linear technology with  $m = u - \gamma$  and  $g = u/2$ , with  $u_0 = 0$ . Equation (22) pins down the inference function here, with  $\hat{u}'(u) = 2/3$ . Figure 1 contrasts the true empirical moment function  $m[u, g(\hat{u}(u))]$  and the econometrician's model-implied moment function  $m(u, \gamma_0)$ . The former accounts for policy feedback and the latter fails to do so. Here the econometrician incorrectly imputes the dampened sensitivity of the observed moment to changes in  $u$  to small changes in  $u$ . Figure 2 shows the resulting single crossing of  $\hat{u}$  with the 45 degree line from above, consistent with the notion of dampened sensitivity. Finally, since  $g$  has here been assumed to be increasing, Figure 3 shows the resulting policy overshooting relative to the optimal policy for low values of  $u$  and undershooting relative to the optimal policy for high values of  $u$ .

Notice, under the conditions stated in Proposition 3, discretionary government policy moves in the right direction relative to the status quo. For example, if  $\gamma(u) > \gamma_0$ ,  $\gamma[\hat{u}(u)] > \gamma_0$ . However, the size of change will be smaller than optimal. In the present example,  $\gamma[\hat{u}(u)] < \gamma(u)$ .

We next consider the nature of inference and policy bias under alternative technologies. However, before doing so, we must establish a sufficient condition for the existence of a well-behaved solution to the inference equation. After all, if we consider departures from the technologies assumed in the preceding proposition, it is possible that there is no solution to the inference equation. To see this, consider the linear technology and suppose that, departing from the preceding two propositions,  $\alpha$  and  $\beta$  have the same sign and  $\kappa > 0$  or  $\alpha$  and  $\beta$  have different signs and  $\kappa > 0$ . In either case, it is possible that  $\alpha = \beta\kappa$  so that there is no solution to the inference equation. With such a possibility in mind, the next lemma provides a sufficient condition for the existence of a continuously-differentiable solution to the inference equation.

**Lemma 2.** *If*

$$m_1(x, \gamma_0) \neq m_2[u, g(x)]g'(x) \quad \forall (x, u) \in \mathbb{R} \times \mathbb{R}, \quad (23)$$

*then there exists a continuously differentiable strictly monotone function  $\hat{u}(\cdot)$  satisfying the inference equation (7) for all  $u \in \mathbb{R}$ .*

**Proof.** Define the following candidate solution to the inference equation

$$\hat{u}(u) \equiv u_0 + \int_{u_0}^u \frac{m_u[v, g(\hat{u}(v))]}{m_u[\hat{u}(v), \gamma_0] - m_\gamma[v, g(\hat{u}(v))]g'[\hat{u}(v)]} dv.$$

Since here  $\hat{u}(u_0) = u_0$ , the candidate solution satisfies the inference equation at  $u_0$  (Proposition 1). Further, under the stated assumptions, the candidate solution has a well-defined derivative at all points, given in equation (18). Rearranging terms in equation (18), it follows that the candidate solution satisfies the differential form of the inference equation (15) point-wise. Thus,  $\hat{u}$  is a continuous and differentiable solution to the inference equation. Moreover,  $\hat{u}$  is continuously differentiable since  $m$  and  $g$  are continuously differentiable. Finally, the sign of the numerator in equation (18) is constant. And the sign of the denominator of this same equation cannot change since, by the Location of Roots Theorem, this would imply the existence of an intermediate point such that the inequality in equation (23) is violated. Thus,  $\hat{u}$  must be strictly monotonic. ■

To take a specific example, if the conditions of Lemma 2 were to be satisfied in the context of the linear technology (equation (20)), then it follows  $\alpha \neq \beta\kappa$  and the linear technology inference function (22) along with its derivative would be well-defined.

We have the following proposition.

**Proposition 4.** *Let  $m_u m_\gamma > 0$  and  $g' > 0$  or let  $m_u m_\gamma < 0$  and  $g' < 0$ , with condition (23) being satisfied. If*

$$\frac{m_\gamma(u_0, \gamma_0)g'(u_0)}{m_u(u_0, \gamma_0)} < 1,$$

*there exists a continuously differentiable strictly monotonic increasing function  $\hat{u}(\cdot)$  satisfying the inference equation. For all  $u < u_0$ ,  $\hat{u}(u) < u$  and for all  $u > u_0$ ,  $\hat{u}(u) > u$ . If  $g$  is increasing then  $u < u_0$  implies  $\gamma^*(u) < g(u)$  and  $u > u_0$  implies  $\gamma^*(u) > g(u)$ . If  $g$  is decreasing then  $u < u_0$  implies  $\gamma^*(u) > g(u)$  and  $u > u_0$  implies  $\gamma^*(u) < g(u)$ .*

*If*

$$\frac{m_\gamma(u_0, \gamma_0)g'(u_0)}{m_u(u_0, \gamma_0)} > 1,$$

*then there exists a continuously differentiable strictly monotonic decreasing function  $\hat{u}(\cdot)$  satisfying the inference equation. For all  $u < u_0$ ,  $\hat{u}(u) > u_0 > u$  and for all  $u > u_0$ ,  $\hat{u}(u) < u_0 < u$ . If  $g$  is increasing then  $u < u_0$  implies  $\gamma^*(u) > \gamma_0 > g(u)$  and  $u > u_0$  implies  $\gamma^*(u) < \gamma_0 < g(u)$ . If  $g$  is decreasing then  $u < u_0$  implies  $\gamma^*(u) < \gamma_0 < g(u)$  and  $u > u_0$  implies  $\gamma^*(u) > \gamma_0 > g(u)$ .*

**Proof.** From Lemma 2 there exists a continuously differentiable strictly monotonic solution to the

inference equation. From the final line in equation (19) it follows

$$\frac{m_\gamma(u_0, \gamma_0)g'(u_0)}{m_u(u_0, \gamma_0)} < 1 \Rightarrow \hat{u}'(u_0) > 1.$$

Considering this case,  $\hat{u}$  must be strictly monotone increasing. Moreover, on the left neighborhood of  $u_0$ ,  $\hat{u}(u) < u$  and on the right neighborhood of  $u_0$ ,  $\hat{u}(u) > u$ . From the continuity of  $\hat{u}$  and Proposition 1 it follows that for all  $u < u_0$ ,  $\hat{u}(u) < u$  and for all  $u > u_0$ ,  $\hat{u}(u) > u$ .

For the second part of the proposition, note that

$$\frac{m_\gamma(u_0, \gamma_0)g'(u_0)}{m_u(u_0, \gamma_0)} > 1 \Rightarrow \hat{u}'(u_0) < 0.$$

Considering this case,  $\hat{u}$  must be strictly monotone decreasing. It follows that for all  $u < u_0$ ,  $\hat{u}(u) > u_0 > u$  and for all  $u > u_0$ ,  $\hat{u}(u) < u_0 < u$ . The clauses pertaining to discretionary government policy follow from the fact that  $\gamma^* = g(\hat{u})$ . ■

Inspection of equation (15) reveals the intuition for the first part of the preceding proposition. Under the posited technologies, the policy feedback effect causes the observed moment to be more sensitive to changes in  $u$  than is understood by the econometrician. She will then incorrectly impute large changes in the moment to large changes in  $u$ . That is,  $\hat{u}$  will tend to have a slope in excess of unity, so that  $\hat{u}$  undershoots for  $u < u_0$  and overshoots for  $u > u_0$ . In other words, the function  $\hat{u}(u)$  will cross the function  $u$  at the point  $u_0$  from below.

These effects are illustrated in Figures 4, 5 and 6 which consider the linear technology with  $m = u + \gamma$  and  $g = u/2$ , with  $u_0 = 0$ . Equation (22) pins down the inference function here, with  $\hat{u}'(u) = 2$ . Figure 4 shows how the econometrician will incorrectly impute large changes in the moment to large changes in  $u$ . Figure 5 shows the resulting single crossing of  $\hat{u}$  with  $u$  from below. Finally, since  $g$  has here been assumed to be increasing, Figure 6 shows the resulting policy undershooting for low values of  $u$  and overshooting for high values of  $u$ .

The second part of the preceding proposition is illustrated most vividly by considering a particular example. To this end, consider the same linear moment  $m = u + \gamma$  but now assume  $g = 2u$ , with  $u_0 = 0$ . That is, in the case being considered, discretionary government policy is relatively sensitive to the inferred value of the structural parameter. Equation (22) pins down the inference function here, with  $\hat{u}'(u) = -1$ . That is,  $\hat{u}(u) = -u$  for all  $u$ . Rather than the parameter estimate simply overshooting and then undershooting, here we have a situation where the inferred value of

the parameter has the wrong sign with probability 1. Of course, this implies that discretionary government policy will move in exactly the opposite direction relative to what is optimal.

Figures 7, 8, and 9 depict the nature of inference under this technology. For example, suppose the realized value is  $u = 5$ . Firms conjecture the econometrician will infer  $\hat{u}(5) = -5$  with the discretionary governmental policy then set to  $\gamma = 2\hat{u} = -10$ . The observed moment will be  $m = u + \gamma = 5 - 10 = -5$ . The econometrician incorrectly believes she is observing  $m = u + 2u_0 = u$  and so indeed draws the inference conjectured by the firms, with  $\hat{u} = -5$ .

#### 4. Joint Estimation and Control under Rational Expectations

This section considers whether and how the econometrician can achieve unbiased parameter inference.

##### 4.1. Avoiding Bias and Achieving Optimality

A natural to ask is whether it is possible to achieve unbiased parameter inference in the setting considered. Introspection suggests a ready solution. The underlying source of biased parameter inference in the preceding section was the failure of the econometrician to parameterize her model in a manner consistent with the rational expectations held by the firms (Assumption 6). Therefore, achieving unbiased inference would seem to necessitate “parameterizing” expectations correctly—with the issue being that the policy expectation is correctly understood as a function, rather than a parameter. Indeed, we have the following lemma.

**Lemma 3.** *If firms anticipate discretionary policy outcomes  $\gamma^{**}(\cdot)$ , then parameter inference will be unbiased for all  $u \in \mathbb{R}$  only if the structural model specifies discretionary policy outcomes as  $\gamma^{**}(\cdot)$ , with resulting rational expectations inference equation*

$$m[u, \gamma^{**}(u)] = m[\hat{u}(u), \gamma^{**}(u)]. \quad (24)$$

**Proof.** Suppose the structural model specifies firm beliefs according to some function  $\tilde{\gamma}(\cdot)$ . Then the inference equation will be

$$m[u, \gamma^{**}(u)] = m[\hat{u}(u), \tilde{\gamma}(\hat{u}(u))]. \quad (25)$$

Thus

$$\hat{u}(u) = u \Rightarrow m[u, \gamma^{**}(u)] = m[u, \tilde{\gamma}(u)] \Rightarrow \tilde{\gamma} = \gamma^{**}. \quad (26)$$

The second implication follows from the strict monotonicity of  $m$  in its second argument. ■

Of course, the government's ultimate objective is not to achieve unbiased parameter inference but rather to implement the optimal policy. Therefore, the government would like to construct a rational expectations equilibrium predicated upon correct inference *and* firms anticipating a specific endogenous outcome

$$\gamma^{**}(\cdot) = g(\cdot).$$

But a necessary condition for correct parameter inference to be feasible for all  $u$  is that the empirical moment be invertible. To this end, let

$$\mu(u) \equiv m[u, g(u)]. \tag{27}$$

We then have the following proposition.

**Proposition 5.** *Let the empirical moment  $\mu(\cdot)$  (equation (27)) be strictly monotone. Then parameter inference will be unbiased for all  $u \in \mathbb{R}$  if and only if the structural model specifies discretionary policy outcomes as  $g(\cdot)$ .*

**Proof.** The “only if” part of the proposition follows from Lemma 3. For sufficiency, suppose the structural model specifies firm beliefs according to some function  $\tilde{\gamma}(\cdot)$ . Then the inference equation will be

$$m[u, g(u)] = m[\hat{u}(u), \tilde{\gamma}(\hat{u}(u))]. \tag{28}$$

For sufficiency, note

$$\tilde{\gamma} = g \Rightarrow m[u, g(u)] = m[\hat{u}(u), g(\hat{u}(u))] \Rightarrow \hat{u}(u) = u. \blacksquare$$

It follows that in order for the econometrician to avoid bias and achieve first best, she must replace the faulty inference equation (7) with the rational expectations inference equation

$$m[u, g(u)] = m[\hat{u}(u), g(u)]. \tag{29}$$

Formally, this inference procedure can be summarized as follows

$$\begin{aligned} \hat{u}^{**}(\cdot) &= m_g^{-1}[m(\cdot, g(\cdot))]. \\ m_g(\cdot) &\equiv m[\cdot, g(\cdot)]. \end{aligned} \tag{30}$$

Of course, the measured agents must understand the econometrician’s procedure. Formally, in a rational expectations equilibrium there is no need for any agent to make a speech. Nevertheless, heuristically, in support of the postulated equilibrium, the econometrician could be understood as making the following speech to the firms.

I the structural econometrician will correctly infer the true value of the parameter  $u$  from the observation of the moment  $m$  that your actions generate in the aggregate. Further, armed with my correct inference, the government will implement the optimal policy  $g(u)$  should it enjoy policy discretion. And now that I have made this speech to you, I know that you know I will do this, and so you should anticipate  $g(u)$  as the discretionary government policy and, thus, act accordingly.

To further aid intuition, it is useful to express the rational expectations inference equation (29) in differential form:

$$m_u[u, g(u)] + m_\gamma[u, g(u)]g'(u) = m_u[\hat{u}(u), g(u)]\hat{u}'(u) + m_\gamma[u, g(u)]g'(u). \quad (31)$$

The left side of the preceding equation reflects how the moment actually changes with  $u$ , and the right side reflects how the structural model treats the moment as changing with  $u$ . The econometrician’s structural model of firm behavior now takes into account firm expectations regarding policy recommendations, while the “counterfactuals” approach failed to do so.

## 4.2. Gallant and Tauchen Revisited

In the title to their important paper, Gallant and Tauchen (1996) pose a question often asked by structural modellers: “Which Moment to Match?” An overarching message of our paper is that the nature of econometric inference changes fundamentally if one is attempting joint estimation and control, rather than simply attempting estimation. This message carries over to moment selection.

To illustrate this, consider an econometrician operating in a world with linear technologies, with two moments being considered candidates for matching. In particular, suppose the optimal government policy is  $\kappa u$ , where moments 1 and 2 have the following forms, respectively:

$$\begin{aligned} m_1 &\equiv \beta_1 \gamma \\ m_2 &\equiv \alpha_2 u + \beta_2 \gamma \\ \alpha_2 &\equiv -\beta_2 \kappa. \end{aligned}$$

According to the traditional moment selection criteria, moment 1 would be discarded since it violates the standard moment monotonicity condition (Assumption 1). In particular, according to the traditional moment selection criteria, moment 1 would be viewed as completely uninformative about the unknown parameter. In contrast, moment 2 would be viewed as informative about the unknown parameter.

But recall, the econometrician is engaged in an exercise of joint estimation and control. With the government attempting to achieve first-best. In this context, moment 1 is highly informative and moment 2 is uninformative. In particular, consider a conjectured rational expectations equilibrium with correct inference and first-best policy implementation. In such an equilibrium the two moments can be expressed as univariate functions of the unknown parameter. We have

$$\begin{aligned}\mu_1 &= \beta_1 \gamma^{**}(u) = \beta_1 g(u) = \beta_1 \kappa u \\ \mu_2 &= \alpha_2 u + \beta_2 \gamma^{**}(u) = \alpha_2 u + \beta_2 g(u) = [\alpha_2 + \beta_2 \kappa] u = 0.\end{aligned}\tag{32}$$

Notice, we have here a situation where without policy feedback, moment 2 is informative and moment 1 is uninformative. Conversely, with policy feedback, moment 2 is uninformative and moment 1 is informative. Strikingly, moment 2 can be highly informative about the true value of the unknown parameter solely due to its sensitivity to the governmental policy variable. Intuitively, as  $u$  changes, so too does governmental policy in equilibrium, and this causes firm behavior to change in a manner informative about  $u$ .

We thus have the following proposition.

**Proposition 6.** *Monotonicity of the moment function  $m(u, \gamma)$  in its first argument (Assumption 1) is neither necessary nor sufficient for  $m$  to be informative about the unknown parameter with joint estimation of  $u$  and control of  $\gamma$ .*

### 4.3. An Algorithmic Approach to Structural Inference

This section considers a small departure from rational expectations' communism of models. In particular, consider the same information structure as above, but assume that, unlike the other agents, the econometrician does not know the government policy function  $g$ . Rather, the econometrician will report her inference  $\hat{u}$  to the government. The government will then "tentatively adopt" a policy  $g(\hat{u})$ . Based on this, the econometrician can reconsider her inference.

Intuitively, the econometrician may not have full knowledge of the government's objectives. However, she should be able to recognize the inconsistency that is the source of biased inference (Section 3): The structural model is predicated upon the assumption that the status quo will be implemented even when the government enjoys policy discretion, and yet the government prefers the policy  $g(\hat{u}) \neq \gamma_0$  based on the inferred value of the parameter.

The question we now address is whether, despite not knowing  $g$ , the econometrician can iterate to (approximately) correct inference of  $u$ , so that the firms will be justified in conjecturing a rational expectations equilibrium in which policy converges to first-best, with  $\gamma^*(u)$  arbitrarily close to  $g(u)$ . To this end, we propose the following *Algorithmic Inference Approach*:

- Start iteration  $n \in \{1, 2, 3, \dots\}$  with parameterized discretionary policy  $\gamma_n$ ;
- Draw inference  $\hat{u}_n$  solving

$$m_{observed} = m(\hat{u}_n, \gamma_n); \tag{33}$$

- Observe the tentative government policy  $g(\hat{u}_n)$  and define this to be  $\gamma_{n+1}$ ;
- Iterate until (approximate) internal consistency,  $|\gamma_{n+1} - \gamma_n| < \epsilon$  for  $\epsilon$  arbitrarily small.

We have the following proposition showing that if  $\mu$  (equation (27)) is strictly monotonic, elimination of internal inconsistency is sufficient to ensure correct inference and optimal government policy.

**Proposition 7.** *Let  $\mu$  (equation (27)) be strictly monotonic. At the  $n$ -th iteration, let the structural model be parameterized assuming government will implement  $\gamma_n$  should it enjoy policy discretion. The resulting inference  $\hat{u}_n$  will be equal to the true parameter  $u$  if and only if  $\hat{u}_n$  rationalizes  $\gamma_n$  so that policy convergence obtains with  $\gamma_n = g(\hat{u}_n) \equiv \gamma_{n+1}$ .*

**Proof.** To establish sufficiency suppose  $\gamma_n = g(\hat{u}_n)$ . Under the stated conditions, the inference equation (7) can be rewritten as

$$m_{observed} = m[\hat{u}_n, g(\hat{u}_n)] \Rightarrow m_{observed} = \mu(\hat{u}_n).$$

From monotonicity of  $\mu$ , the unique value at which the observed moment matches the model-implied moment is the true  $u$ . To establish necessity, suppose  $\gamma_n \neq g(\hat{u}_n)$ . It then follows from the moment

matching equation and monotonicity of  $m$  in its second argument that

$$m_{observed} = m(\hat{u}_n, \gamma_n) \neq m[\hat{u}_n, g(\hat{u}_n)] \equiv \mu(\hat{u}_n).$$

Since  $m_{observed} \neq \mu(\hat{u}_n)$  it follows  $\hat{u}_n \neq u$ . ■

Of course, in practice, iteration will generally continue until approximate convergence. Therefore, it is interesting to evaluate the convergence properties of the preceding algorithm. Rather than do so numerically with arbitrary examples, we first consider below iterating on the preceding algorithm in the case of the linear technology. To begin, note that iterating on  $\gamma_n$  values is equivalent to iterating on the  $u$  values that would justify them, e.g.  $\kappa u_{n+1} \equiv \gamma_{n+1}$ . Thus, from the statement of the algorithm:

$$\kappa u_{n+1} \equiv \gamma_{n+1} = \kappa \hat{u}_n \Rightarrow u_{n+1} = \hat{u}_n.$$

In the posited rational expectations equilibrium, with first-best policy conjectured by the firms, the inference equation at iteration  $n + 1$  is

$$m[u, g(u)] = m[\hat{u}_{n+1}, \gamma_{n+1}]. \quad (34)$$

With the linear technology, the preceding equation can be expressed as follows

$$\alpha u + \beta \kappa u = \alpha \hat{u}_{n+1} + \beta \kappa \hat{u}_n. \quad (35)$$

Iterating on the preceding equation we have the following lemma which shows that the proposed algorithm will converge to the truth provided the policy feedback effect is sufficiently weak relative to the direct effect.

**Lemma 4.** *Under the linear technology (equation (20)), the Algorithmic Inference Approach yields inference at the  $n$ -th iteration equal to*

$$\hat{u}_n = u + \left(-\frac{\beta \kappa}{\alpha}\right)^n (u_1 - u). \quad (36)$$

*The algorithm converges to the true parameter  $u$  for all  $u \in \mathbb{R}$  for all starting points  $u_1 \in \mathbb{R}$  if and only if*

$$\left|\frac{\beta \kappa}{\alpha}\right| < 1.$$

In fact, Lemma 4 is a special case of a more general convergence condition which relies on bounding the policy feedback effect, as we show next.

**Proposition 8.** *The Algorithmic Inference converges to the true parameter  $u$  for all  $u \in \mathbb{R}$  for all starting points  $\gamma_1 \in \Gamma$  if*

$$\left| \frac{m_\gamma g'}{m_u} \right| < 1.$$

**Proof.** The inference equation is

$$m[u, g(u)] - m[\hat{u}_n, \gamma_n] = 0.$$

The preceding equation can be rewritten as

$$\{m[u, g(u)] - m[\hat{u}_n, g(u)]\} + \{m[\hat{u}_n, g(u)] - m[\hat{u}_n, \gamma_n]\} = 0.$$

From the mean value theorem, for each iteration  $n$ , there exists  $x_n$  between  $\hat{u}_n$  and  $u$ , and there exists  $g_n$  between  $g(u)$  and  $\gamma_n$  such that

$$m_u[x_n, g(u)](u - \hat{u}_n) + m_\gamma(\hat{u}_n, g_n)[g(u) - g(\hat{u}_{n-1})] = 0$$

Applying the mean value theorem to the final term in the preceding equation, we know that for each iteration  $n$  there exists  $z_n$  between  $u$  and  $\hat{u}_{n-1}$  such that

$$m_u[x_n, g(u)](u - \hat{u}_n) + m_\gamma(\hat{u}_n, g_n)g'(z_n)(u - \hat{u}_{n-1}) = 0.$$

Rearranging terms in the preceding equation, we find that at each iteration  $n$

$$u - \hat{u}_n = -\frac{m_\gamma[\hat{u}_n, g_n]g'(z_n)}{m_u[x_n, g(u)]}(u - \hat{u}_{n-1}).$$

Under the stated condition  $\hat{u}_n$  converges to  $u$ . ■

## 5. Quantitative Example

This section considers an econometrician seeking to estimate unobserved costs of corporate bankruptcy based upon the financial policies adopted by corporations. Understanding the magnitude of bankruptcy costs is important for a number of reasons. First, to the extent that bankruptcy costs are deadweight losses, rather than transfers, their magnitude is directly relevant for assessing the efficiency costs of corporate leverage, as well as tax-induced leverage increases. For example,

in making the case for the Bush Administration Treasury for integration of the individual and corporate tax systems, Hubbard (1993) contended, “tax-induced distortions in corporations’ comparisons of nontax advantages and disadvantages of debt entail significant efficiency costs.” Second, the magnitude of bankruptcy costs is indirectly relevant to the tax authority estimating revenues. After all, higher bankruptcy costs serve as a counterweight to tax benefits of debt, discouraging firms from taking on extremely high leverage. For example, Gruber and Rauh (2007) estimate the tax elasticity of corporate income is only -0.2, evidence that would appear to contradict Hubbard’s notion that corporations aggressively change capital structures in response to tax incentives.

Early models, such as that of Stiglitz (1973), failed to deliver interior optimal leverage ratios. Lacking interior optimal leverage ratios, computational general equilibrium (CGE) models, e.g. Ballard, et. al (1985), posited exogenous financing rules. In the absence of closed models, public finance economists such as Gordon and MacKie-Mason (1990) and Nadeau (1993) were forced into positing ad hoc costs of financial distress. In an important contribution, Leland (1994) showed how to develop a tractable logically closed model of capital structure for firms facing taxation and costs of distress using contingent-claims pricing methods.

In this section, we use Leland’s canonical framework to understand the nature of bias that can arise in careful structural work in a real-world applied policy setting. Consider a government that is interested in setting the corporate income tax in a way that is optimal according to its objective function. The magnitude of financial distress costs is clearly relevant here since, as argued above, the magnitude of these costs determines efficiency costs of corporate leverage, as well as having a direct bearing on tax collections.

With this economic setting in mind, consider a structural econometrician who will observe the financing policies adopted by a set of homogeneous firms funding new investments during the pre-inference stage.<sup>8</sup> Specifically, the econometrician will measure the mean interest coverage ratio, as measured by the ratio of EBIT to interest expense. As shown below, this moment is directly informative about bankruptcy costs.

Consider first the decision problem of the firms. Each firm will choose a promised instantaneous coupon on a consol bond, denoted  $\phi$ . The firm will use the debt proceeds plus equity injections to

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<sup>8</sup>Firms can have differing EBIT levels. The optimal coupon is linear in EBIT so coverage ratios will be equal nevertheless.

fund a new investment, as is standard in project finance settings. We assume parameters are such that the investment has positive net present value. Formally, the new investment has positive net present value if the value of the levered enterprise exceeds the cost of the investment.

Debt enjoys a tax advantage, with interest being a deductible expense on the corporate income tax return. Consequently, each instant it is alive, the project firm will capture a gross tax shield equal to  $\phi\tilde{\gamma}$ , with the random variable  $\tilde{\gamma}$  now representing the corporate income tax rate that will be implemented just after the econometrician completes her parameter inference. However, debt service has a negative impact on the firm's value. In particular, in the event of EBIT being insufficient to service the coupon, the firm's debt will be cancelled and bondholders will capture a fraction  $N(u)$  of unlevered firm value. The function  $N$  here is the standard normal cumulative distribution function.

Finally, suppose EBIT follows a geometric Brownian motion with drift  $\mu$ , volatility  $\sigma$ , and initial value normalized at 1. The risk-free rate is denoted  $r$ . The objective is to maximize levered project value. Or equivalently, firms maximize expected tax shield value minus expected default costs. Thus, firms solve the following program

$$\max_{\phi} \quad \gamma\phi \left(\frac{1}{r}\right) (1 - \phi^{-\lambda}) - N(u) \frac{\phi(1 - \gamma)}{r - \mu} \phi^{-\lambda}. \quad (37)$$

where  $\lambda$  is the negative root of the following quadratic equation

$$\frac{1}{2}\sigma^2\lambda^2 + \left(\mu - \frac{1}{2}\sigma^2\right)\lambda - r = 0. \quad (38)$$

Note, the first term in the objective function captures tax shield value and the second term captures bankruptcy costs. Effectively, the tax shield represents an annuity that expires at the first passage of EBIT to the coupon from above. At this same point in time, bankruptcy costs incurred. This explains the presence of the term  $\phi^{-\lambda}$  in the objective function, which measures the price at date zero of a so-called primitive claim paying 1 the first passage of EBIT to the coupon from above.

The first-order condition for the optimal coupon entails equating marginal tax benefits with marginal bankruptcy costs. In particular, the optimal coupon satisfies

$$\left(\frac{\gamma}{r}\right) [1 - (1 - \lambda)\phi^{-\lambda}] = (1 - \lambda)N(u) \frac{(1 - \gamma)}{r - \mu} \phi^{-\lambda}. \quad (39)$$

Rearranging terms in the preceding equation, it follows the optimal coupon is

$$\phi^* = (1 - \lambda)^{1/\lambda} \left[ 1 + N(u) \frac{(1 - \gamma) r}{r - \mu \gamma} \right]^{1/\lambda}. \quad (40)$$

The moment observed by the econometrician, the mean interest coverage ratio, is  $1/\phi^*$ . Thus, in the present setting

$$m(u, \gamma) \equiv \mathbb{E}[\phi^{-1}] = (1 - \lambda)^{-1/\lambda} \left[ 1 + N(u) \frac{(1 - \gamma) r}{r - \mu \gamma} \right]^{-1/\lambda}. \quad (41)$$

Notice, in this particular case,  $m_u(u, \gamma) > 0$  and  $m_\gamma(u, \gamma) < 0$ . That is, the optimal interest coverage ratio is increasing in bankruptcy costs and decreasing in the tax rate.

Suppose now that the structural econometrician, who recommended the Trump tax cut, failed to impose the assumption that firms have rational expectations regarding the tax rate. Specifically, suppose the econometrician treated the tax change as a counterfactual event and parameterized her model using the status quo tax rate. In the present context, the inference equation (7) takes the form

$$\begin{aligned} m[u, \gamma^*(u)] &= (1 - \lambda)^{-1/\lambda} \left[ 1 + N(u) \frac{[1 - \gamma^*(u)] r}{r - \mu \gamma^*(u)} \right]^{-1/\lambda} \\ &= (1 - \lambda)^{-1/\lambda} \left[ 1 + N(\hat{u}) \frac{(1 - \gamma_0) r}{r - \mu \gamma_0} \right]^{-1/\lambda} = m(\hat{u}, \gamma_0). \end{aligned} \quad (42)$$

Cancelling terms in the preceding equation and solving one obtains

$$N(\hat{u}) = \frac{[1 - \gamma^*(u)]/\gamma^*(u)}{(1 - \gamma_0)/\gamma_0} \times N(u). \quad (43)$$

How important quantitatively is the bias implied by the preceding equation? To assess this, we must stipulate parameter values. Following Goldstein, Ju and Leland (2001) we approximate the effect of personal taxes by setting  $\gamma$  equal to the Miller (1977) debt tax shield value. In particular, let  $\gamma_c$  denote the corporate tax rate,  $\gamma_e$  denote the equityholder tax rate, and  $\gamma_d$  denote the debtholder tax rate. The Miller debt tax shield value is

$$\gamma = 1 - \frac{(1 - \gamma_c)(1 - \gamma_e)}{(1 - \gamma_d)}. \quad (44)$$

Goldstein, Ju and Leland (2001) assume  $\gamma_c = 35\%$ ,  $\gamma_e = 20\%$  and  $\gamma_d = 35\%$ . These parameter values are reflective of the status quo before the Trump corporate tax cut, which implies the status quo policy value is  $\gamma_0 = 20\%$ . The Trump tax reform cut the corporate income tax rate to  $\gamma_c = 21\%$ . This tax rate reduction substantially lowered the effective debt tax shield to  $\gamma = 2.8\%$ . Substituting these values into the bias formula in equation (43) we find

$$N(\hat{u}) = 8.68 \times N(u).$$

That is, estimated bankruptcy costs here are 8.68 times actual bankruptcy costs. Intuitively, here the firms choose low leverage in rational anticipation of the upcoming tax cut. The econometrician treats the firms as ignorant of the prospective tax cut and treats the low leverage as indicative of very high bankruptcy costs.

Figure 10 illustrates the cause of the biased inference. The two schedules plot the interest coverage ratio adopted by firms for alternative values of bankruptcy costs. The schedules differ according to the assumption adopted in the structural model regarding the tax rate. In one case, the structural model is predicated upon the assumption that agents expect the status quo to be maintained. In the other case, agents have a rational expectation of the actual policy change, a large corporate tax cut, leading them to adopt low leverage. The failure to account for rational expectations would cause the econometrician to impute the observed low leverage to high bankruptcy costs.

Such biased parameter estimates will lead to faulty predictions regarding the behavior of firms after the policy change and a faulty assessment of policy tradeoffs. Continuing to follow the parameterization of Goldstein, Ju and Leland, assume  $r = 4.5\%$ ,  $\sigma = 0.25$ ,  $\mu = 0$ , and  $N(u) = 5\%$ . Evaluated at this parameterization, with the equilibrium  $\gamma^* = 2.8\%$ , equation (40) implies firms observed during the inference stage will choose coupons equal to 13.63% times initial EBIT(=1). And note, future generations of firms will adopt this same coupon rate. After all, under rational expectations, the inference stage firms here will posit the same tax shield value as that which will actually be operative post-inference. In other words, no reaction will be apparent when one contrasts the behavior of the inference-stage firms with the behavior of firms post-inference.

The structural econometrician will here mistakenly predict that future generations of firms will respond to the tax rate change by adopting a much lower coupon rate, failing to understand that the inference-stage firms already responded rationally to the upcoming change. In particular, based upon an estimated bankruptcy cost equal to 43.4%(= 8.68  $\times$  5%), equation (40) leads to a predicted coupon rate, call it  $\hat{\phi}$ , equal to only 1.49% times initial EBIT. However, as shown above, the actual coupon rate after the tax rate change will be 13.63% times EBIT.

The faulty parameter inference leads to faulty predictions regarding firm behavior after the policy change which in turn leads to a faulty assessment of policy tradeoffs. To illustrate, note that the present value of tax collections per firm in this economy is equal to the value of the perpetual

stream of taxes on an unlevered entity minus the tax shield value. It follows that the actual and predicted present value of tax collections are, respectively

$$\begin{aligned} T &= \frac{1-\gamma}{r-\mu} - \gamma\phi^* \left(\frac{1}{r}\right) [1 - (\phi^*)^{-\lambda}] = .5546 \\ \hat{T} &= \frac{1-\gamma}{r-\mu} - \gamma\hat{\phi} \left(\frac{1}{r}\right) [1 - \hat{\phi}^{-\lambda}] = .6133 \end{aligned}$$

That is, the actual present value of tax collections here will be 10.6% lower than predicted tax collections. Intuitively, the upward bias in estimated bankruptcy costs leads to a faulty prediction of low leverage leading to a faulty prediction of high corporate income tax collections.

## 6. Multivariate Extension

The preceding sections considered an econometrician attempting to infer one unknown variable, with government policy also being univariate. In this section, we consider a multivariate extension. For simplicity, linearity is assumed.

There are  $n_u \geq 1$  unknown variables, each with support on the real line, with the realized vector denoted  $\mathbf{u}$ . The econometrician seeks to infer  $\mathbf{u}$  based upon a vector  $\mathbf{m}$  consisting of  $n_u$  empirical moments. The government has  $n_\gamma \geq 1$  policy tools, with the full-information optimal policy being  $\mathbf{g}(\mathbf{u})$ .

The observed empirical moments are linear:

$$\mathbf{m} \equiv \mathbf{A}\mathbf{u} + \mathbf{B}\boldsymbol{\gamma}.$$

In the preceding equation,  $\mathbf{A}$  is an  $n_u \times n_u$  matrix with element  $\alpha_{ij}$  denoting the moment  $i$  coefficient on variable  $u_j$ . Matrix  $\mathbf{B}$  is an  $n_u \times n_\gamma$  matrix with element  $\beta_{ij}$  denoting the moment  $i$  coefficient on government policy variable  $\gamma_j$ . The government policy vector is:

$$\boldsymbol{\gamma} = \mathbf{K}\hat{\mathbf{u}}.$$

In the preceding equation,  $\mathbf{K}$  is an  $n_\gamma \times n_u$  matrix, with element  $\kappa_{ij}$  denoting the policy  $i$  coefficient on  $\hat{u}_j$ .

Consider again the nature of bias that arises if the econometrician parameterizes government policy at the status quo

$$\boldsymbol{\gamma}_0 \equiv \mathbf{K}\mathbf{u}_0.$$

The inference equation is

$$\mathbf{A}\mathbf{u} + \mathbf{BK}\hat{\mathbf{u}} = \mathbf{A}\hat{\mathbf{u}} + \mathbf{BK}\mathbf{u}_0. \quad (45)$$

The left side of the preceding equation is the observed moment and the right side is the econometrician's structural model. Solving the preceding equation we obtain the multivariate analog of equation (22):

$$\begin{aligned} \hat{\mathbf{u}} &= \mathbf{u} + [\mathbf{A} - \mathbf{BK}]^{-1}\mathbf{BK}[\mathbf{u} - \mathbf{u}_0] \\ &= \mathbf{u} + [\mathbf{A} - \mathbf{BK}]^{-1}\mathbf{B}[\mathbf{g}(\mathbf{u}) - \gamma_0]. \end{aligned} \quad (46)$$

From the preceding equation it follows that

$$\underbrace{\mathbf{g}(\mathbf{u}) = \gamma_0}_{\mathbf{u}=\mathbf{u}_0} \Rightarrow \hat{\mathbf{u}} = \mathbf{u}. \quad (47)$$

However,

$$\hat{\mathbf{u}} = \mathbf{u} \not\Rightarrow \underbrace{\mathbf{g}(\mathbf{u}) = \gamma_0}_{\mathbf{u}=\mathbf{u}_0}. \quad (48)$$

Thus in contrast to Proposition 1, in a multivariate setting  $\mathbf{u} = \mathbf{u}_0$  is sufficient to ensure absence of bias, but is not necessary.

Other implications of the linear multivariate bias equation (46) are most readily illustrated by considering the simplest case with two unknowns and one government policy variable. In this case, let  $\beta_i$  denote the moment  $i$  coefficient on the government policy variable and let  $\kappa_j$  denote the government policy variable coefficient on  $\hat{u}_j$ . Applying equation (46) here we obtain:

$$\begin{aligned} \hat{u}_1 &= u_1 + \frac{[\beta_1 - \beta_2\alpha_{12}/\alpha_{22}][\kappa_1(u_1 - u_{10}) + \kappa_2(u_2 - u_{20})]}{\alpha_{11} - \beta_1\kappa_1 + [\beta_2\kappa_1\alpha_{12} + \beta_1\kappa_2\alpha_{21} - \beta_2\kappa_2\alpha_{11} - \alpha_{12}\alpha_{21}]/\alpha_{22}} \\ \hat{u}_2 &= u_2 + \frac{[\beta_2 - \beta_1\alpha_{21}/\alpha_{11}][\kappa_1(u_1 - u_{10}) + \kappa_2(u_2 - u_{20})]}{\alpha_{22} - \beta_2\kappa_2 + [\beta_2\kappa_1\alpha_{12} + \beta_1\kappa_2\alpha_{21} - \beta_1\kappa_1\alpha_{22} - \alpha_{12}\alpha_{21}]/\alpha_{11}}. \end{aligned} \quad (49)$$

To begin, suppose  $\alpha_{12} = \alpha_{21} = 0$ . The traditional Jacobian formulation would suggest that the problem of inferring  $u_1$  is separable from the problem of inferring  $u_2$ . However, with policy feedback, it is apparent that the inference problems and biases are linked, with

$$\begin{aligned} \hat{u}_1 &= u_1 + \frac{\beta_1[\kappa_1(u_1 - u_{10}) + \kappa_2(u_2 - u_{20})]}{\alpha_{11} - \beta_1\kappa_1 - \beta_2\kappa_2/\alpha_{22}} \\ \hat{u}_2 &= u_2 + \frac{\beta_2[\kappa_1(u_1 - u_{10}) + \kappa_2(u_2 - u_{20})]}{\alpha_{22} - \beta_2\kappa_2 - \beta_1\kappa_1\alpha_{22}/\alpha_{11}}. \end{aligned} \quad (50)$$

Next, recall that in the univariate case, policy was assumed to affect the observed moment, and the inferred value of the unknown parameter was assumed to influence policy, leading to bias. However, the multivariate setting illustrates that an unknown parameter need not enter the government policy  $\mathbf{g}$  function in order for its respective estimate to be biased. To see this, suppose once again that  $\alpha_{12} = \alpha_{21} = 0$ , and suppose further that government policy is independent of  $\hat{u}_2$ , with  $\kappa_2 = 0$ . We then have

$$\begin{aligned}\hat{u}_1 &= u_1 + \frac{\beta_1 \kappa_1 (u_1 - u_{10})}{\alpha_{11} - \beta_1 \kappa_1} \\ \hat{u}_2 &= u_2 + \frac{\beta_2 \kappa_1 (u_1 - u_{10})}{\alpha_{22} - \beta_1 \kappa_1 \alpha_{22} / \alpha_{11}}.\end{aligned}\tag{51}$$

That is, even though  $\hat{u}_2$  does not inform policy,  $\hat{u}_2$  will nevertheless be biased if government policy influences ( $\beta_2 \neq 0$ ) the respective moment (here  $m_2$ ) that is relied upon for inference.

It is also apparent that, in general, the existence of a moment that is independent of policy does not imply the absence of bias in any particular parameter estimate. To see this, suppose all four elements of matrix  $\mathbf{A}$  are positive. Suppose further that government policy does not enter one of the moments, say  $m_2$ , with  $\beta_2 = 0$ . In this case, we have

$$\begin{aligned}\hat{u}_1 &= u_1 + \frac{\beta_1 [\kappa_1 (u_1 - u_{10}) + \kappa_2 (u_2 - u_{20})]}{\alpha_{11} - \beta_1 \kappa_1 + [\beta_1 \kappa_2 \alpha_{21} - \alpha_{12} \alpha_{21}] / \alpha_{22}} \\ \hat{u}_2 &= u_2 + \frac{-[\beta_1 \alpha_{21} / \alpha_{11}] [\kappa_1 (u_1 - u_{10}) + \kappa_2 (u_2 - u_{20})]}{\alpha_{22} + [\beta_1 \kappa_2 \alpha_{21} - \beta_1 \kappa_1 \alpha_{22} - \alpha_{12} \alpha_{21}] / \alpha_{11}}.\end{aligned}\tag{52}$$

## Conclusion

An asserted advantage of moment-based structural microeconometrics over reduced-form methods is that one can correctly identify policy-invariant parameters so that alternative policy options can be assessed. As we have shown, this approach, which generally treats policy changes as counterfactual zero probability events, violates rational expectations: agents inside the structural model should understand that policy changes are positive probability events which the econometric exercise is intended to inform. We examined the implications of this violation of rational expectations in moment-based microeconomic parameter inference which serves a policy function. As shown, bias emerges unless the true value of the parameter justifies the status quo. If instead a policy

change is justified, biased inference occurs. Finally, it was shown how rational expectations can be imposed in an internally consistent manner, yielding unbiased inference and optimal policy.

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Figure 1: Actual Moment versus Model Moment  
Case:  $m(+,-)$ ,  $g(+)$

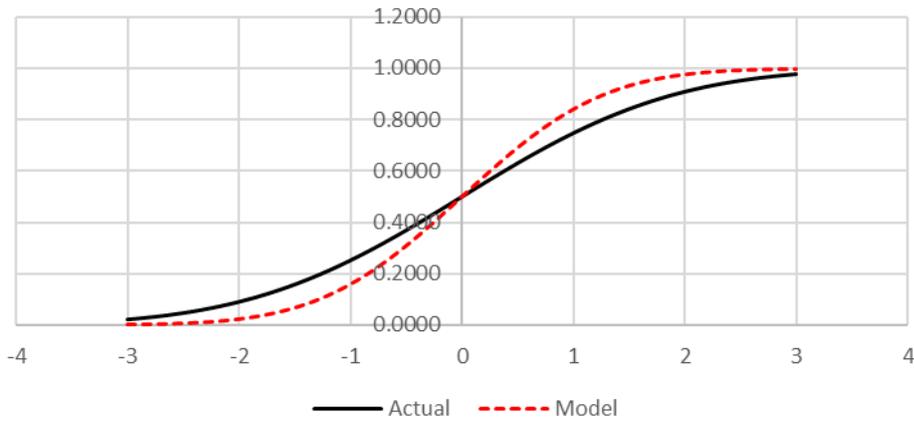


Figure 2: Parameter Inference  
Case:  $m(+,-)$ ,  $g(+)$

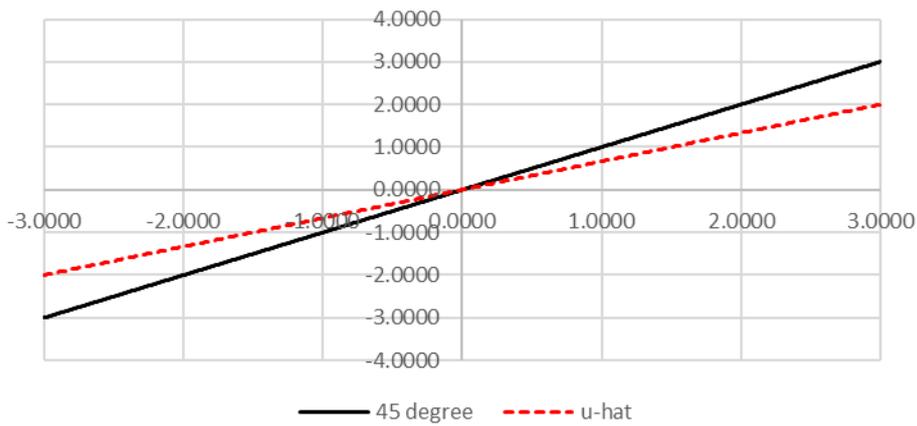


Figure 3: Actual versus Optimal Policy  
Case:  $m(+,-)$ ,  $g(+)$

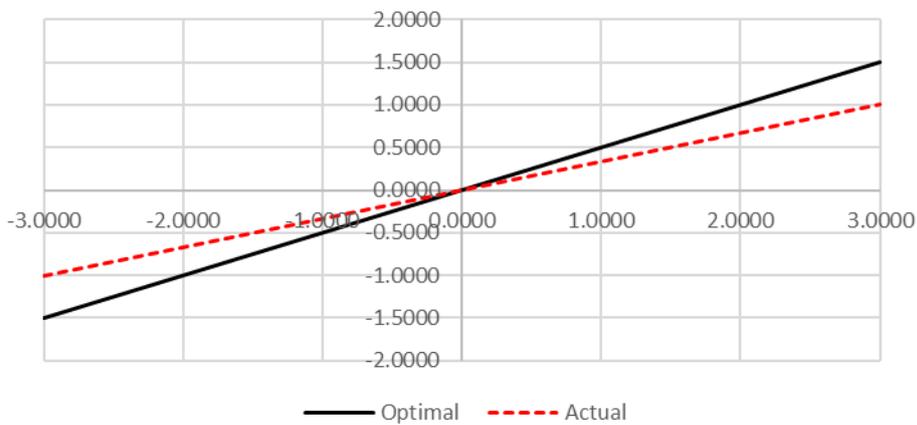


Figure 4: Actual Moment versus Model Moment  
Case:  $m(+,+)$ ,  $g(+)$

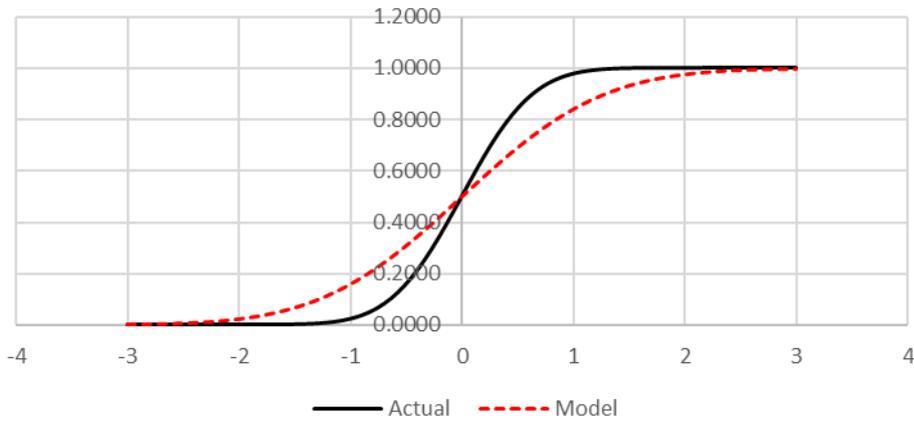


Figure 5: Parameter Inference  
Case:  $m(+,+)$ ,  $g(+)$

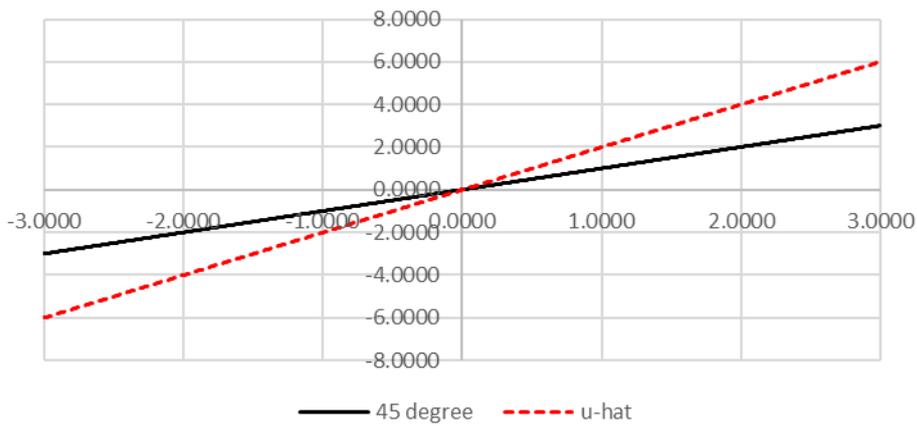


Figure 6: Actual versus Optimal Policy  
Case:  $m(+,+)$ ,  $g(+)$

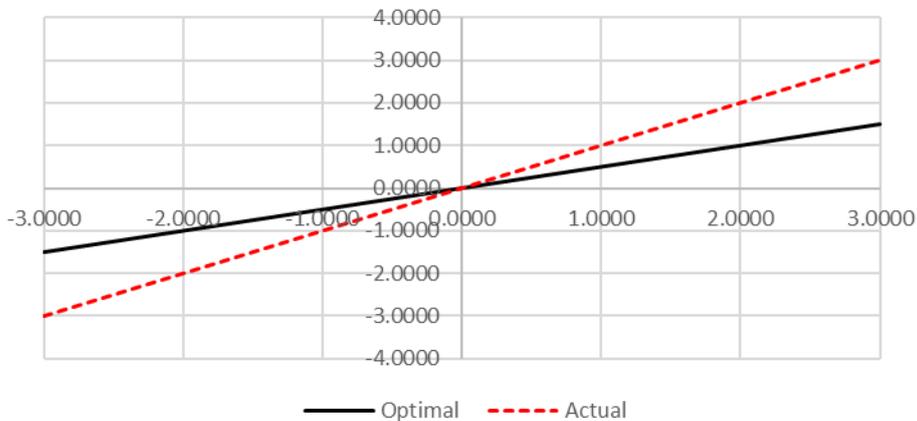


Figure 7: Actual Moment versus Model Moment  
Case:  $m(+,+)$ ,  $g(+)$

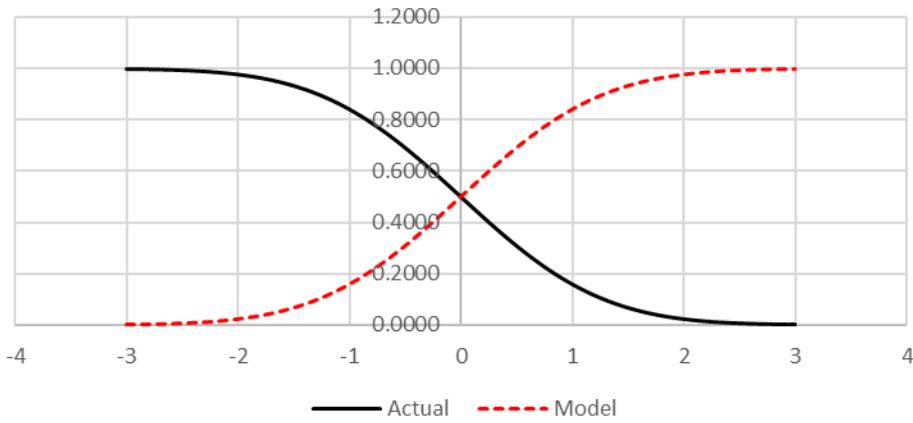


Figure 8: Parameter Inference  
Case:  $m(+,+)$ ,  $g(+)$

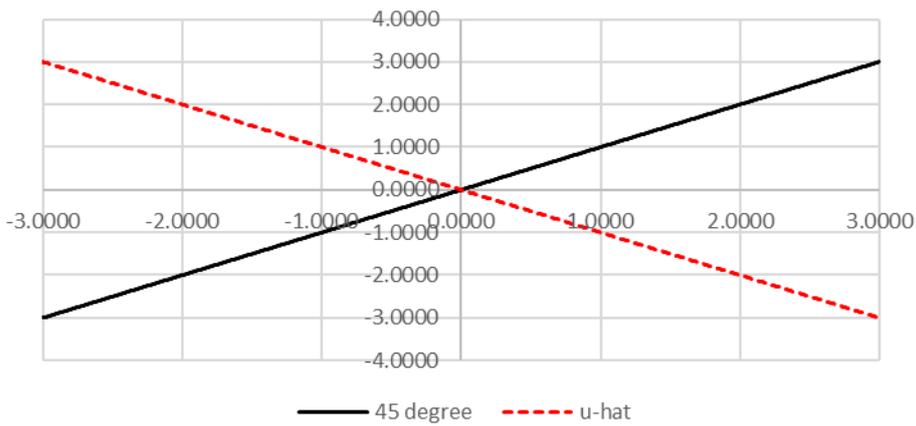


Figure 9: Actual versus Optimal Policy  
Case:  $m(+,+)$ ,  $g(+)$

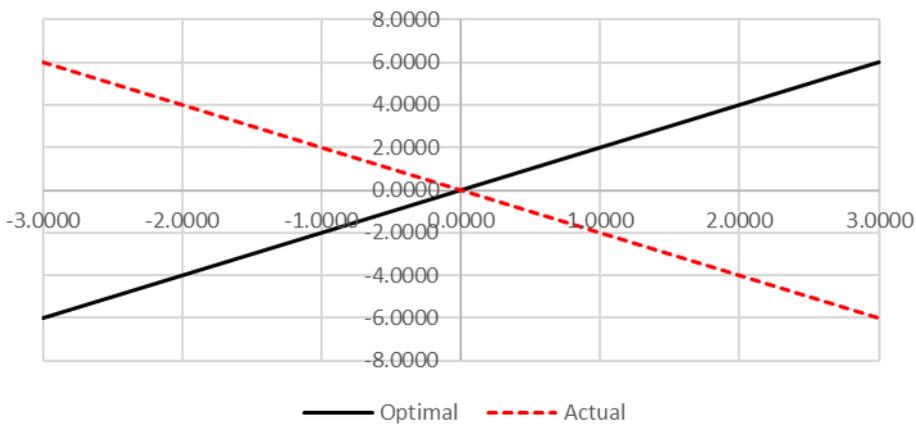


Figure 10: Interest Coverage and Distress Costs

